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BER Performance of Frequency Selective Channels with Cyclic Prefix Based Equalizers

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ABSTRACT

The BER (Bit Error Rate) of the frequency selective channels is analyzed using OFDM (Ortogonal Frequency Division Multiplexing) technique, and MDPSK modulation. The equalization technique is based on cyclic prefix added to the information block. We considered channels with a gaussian noise and Rayleigh or Rice fading conditions.

I. INTRODUCTION

Mobile radio communication systems offer a variety of services and qualities for mobile users. Then, the modern mobile radio transceivers must be able to provide high capacity and variable bit rate (VBR) information transmission with high bandwidth efficiency.

In the radio channel, signals are usually affected by fading and multipath delay spread phenomenon. Severe fading of the signal amplitude and intersymbol interference (ISI) due to the frequency selectivity of the channel cause an unacceptable degradation of the system performance.

The techniques used in the single carrier mobile communication systems -channel coding and adaptive equalization- to combat fading and multipath propagation are practical difficult to use at high bit rate, due to the delay in coding and equalization process and high cost of hardware.

The multicarrier modulation technique, commonly referred as OFDM is a main modulation scheme for digital audio and video broadcasting to avoid ISI introduced by frequency selective (FS) multipath fading. High-rate data are sent in parallel on a number of narrowband ISI free subchannels, each subchannel operating at a low data rate, to avoid the channel frequency selectivity.

Adding a cyclic prefix for each block of data, the ISI can almost completely be avoided. Each carrier of OFDM signals is modulated with M-ary differential phase shift keying (MDPSK).

In this paper we theoretically study the BER performance of OFDM-MDPSK in a frequency selective Rician fading channel, rarely reported in the literature.

Techniques for channel equalization based on redundant filter bank precoder have been introduced and well-situated in recent years [SGB `99, XIA `97]. The cyclic prefix system with DFT (Discrete Fourier Transform) matrices, which is commonly employed in discrete multitone (DMT) systems for twisted pair channels in telephone cables [BIN `90] can actually be used for blind equalization of a much broader class of channels. One advantage of the method is that besides FFT, there is very little computation involved, and therefore the method is very efficient.

If a channel has zero in $|z| \geq 1$ domain then there are some problems with traditional equalization: the channel noise can get severely be amplified. The cyclic prefix method does not require the channel to be minimum-phase. Equalization does not severely amplify noise as long as the zeros of the channel are not too close to the unit circle. The advantages are obtained at the expense of a slightly higher bandwidth expansion ratio compared to [GSB`97], but the expansion becomes negligible as the block length increases.

II. CYCLIC PREFIX

We assume that the channel is an L-th order FIR system:

$$C(z) = \sum_{n=0}^{L} c(n) z^{-n} \qquad (1)$$

with additive noise (fig. 1).

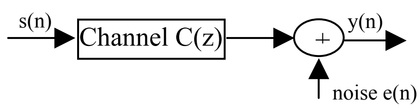

Fig.1. The FIR channel under consideration

The symbol stream is divided into block of length M, and after that, L zeros are introduced at the beginning of each block like a guard interval. The all M+L sequence occupies the same time like original block for a given symbol rate zero-prefix reduce the spacing between samples (fig. 2).

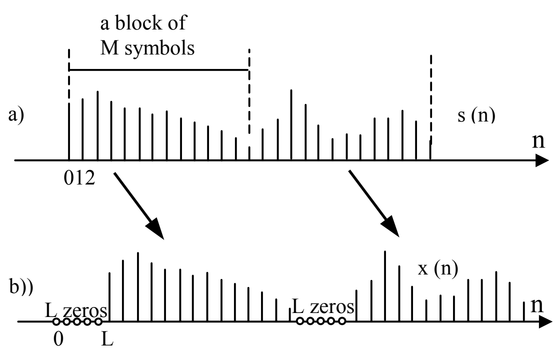

Figure 2.(a) Symbol stream, (b) zero-prefixed version

The bandwidth expansion factor $\gamma = (M+L)/M$ represents the excess bandwidth. By making M large enough, it can reduce γ. From the measurement of output block - that depends on the input and the noise - it can recover the corresponding input block (fig. 2). Ignoring noise for the moment, we have for k-th block:

$$\begin{bmatrix} y(J_k) \\ y(J_{k+1}) \\ y(J_{k+M-1}) \end{bmatrix} = \mathbf{C}_\Delta \begin{bmatrix} s(J_k) \\ s(J_{k+1}) \\ s(J_{k+M-1}) \end{bmatrix} \qquad (2)$$

where J=k(L+M)+L, M>0, and

$$\mathbf{C}_\Delta = \begin{pmatrix} c(0) & 0 & \cdots & 0 \\ c(1) & c(2) & \cdots & 0 \\ c(M-1) & c(M-2) & \cdots & c(0) \end{pmatrix}$$

This is a lower triangular Toeplitz matrix representing causal convolution. Assume $c(0) \neq 0$ (it can extract delays from C(z) if it is necessary). Then $\mathbf{C}_\Delta$ is nonsingular and its inverse $\mathbf{C}_\Delta^{-1}$ is also a lower triangular Toeplitz matrix, like $\mathbf{C}_\Delta$, having the terms h(n) the first M coefficients of the inverse:

$$\frac{1}{C(z)} = \sum_{n=0}^{\infty} h(n) z^{-n} \qquad (3)$$

In practice the channel adds the noise so y(n) $\rightarrow$ y(n) + e(n) in the former relations and so x(n) is not recovered exactly. This noise could be severely amplified by the inversion process if C(z) has some zeros outside the unit circle.

For the case of blind identification the channel order L is known but C(z) itself is unknown. It is still possible to recover the input stream from the output under some conditions. Instead of L zeros it inserts 2L+1 redundant samples (fig.3); it has 2L zeros and an impulse at the middle. It is called "the impulse redundancy" [VV-2]. If this new symbol stream is convolved with the L-th order channel, the impulse response c(n) appears at the output starting from the location of the redundant impulse in each block. Thus it can recover the channel exactly and then invert it to recover the input x(n). The method needs 2L+1 rather L redundant symbols but is computationally and conceptually very simple. The bandwidth expansion factor (2L+1+M)/M tends to unity for large M.

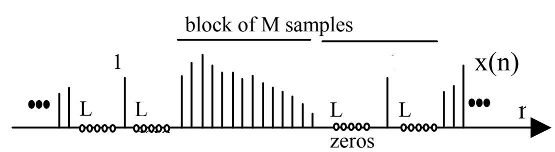

Figure 3. Impulse redundancy of length 2L+1 added to every length-M block of the input symbol stream

The channel noise that affects the redundant impulse can be reduced by averaging the estimation of c(n) over many blocks; this implies a corresponding large latency. The inverse matrix $\mathbf{C}_\Delta^{-1}$ could still amplify noise if the channel is not minimum-phase.

Instead of using a zero-prefix or impulse prefix it can also use a cyclic prefix and perform blind equalization; this method performs even if the channel is not a minimum phase. It is sufficient, but not necessary that the channel be free from unit circles zeros [VV-2]. The L symbols at the end of each block are copied into the beginning of that block, to form the cyclic prefix (fig. 4).

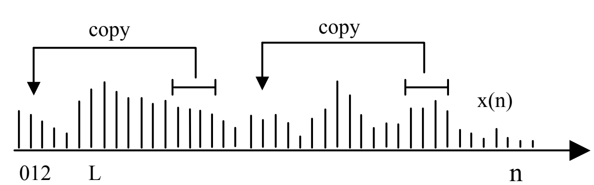

Figure 4. Explanation of how cyclic prefix is inserted

This assume L≤M; if L>M, the matrix becomes circulant and eq. (2) becomes (4). The last input symbols x(n) in the m-th block are related to the last M output symbol y(m) in the m-th block.

$$\mathbf{y(m)= Cs(m)} \qquad (4)$$

where:

s(m)=[s(mM), s(mM+1)…s(mM+M-1)]
y(m)=[y(m(L+M)),y(m(L+M)+1)…
……y(m(L+M)+L-1)],

C is a circulant matrix with the element of top row coming from the channel impulse response c(n); for example when the channel order is L=3 and M=6.

$$\mathbf{C} = \begin{pmatrix} c(0) & 0 & 0 & c(3) & c(2) & c(1) \\ c(1) & c(0) & 0 & 0 & c(3) & c(2) \\ c(2) & c(1) & c(0) & 0 & 0 & c(3) \\ c(3) & c(2) & c(2) & c(0) & 0 & 0 \\ 0 & c(3) & c(2) & c(1) & c(0) & 0 \\ 0 & 0 & c(3) & c(2) & c(1) & c(0) \end{pmatrix}$$

If the channel is known, it can perform the equalization by inverting (4), assuming **C** is nonsingular [VV-1]. The eigenvalues of the M x M circulant are equal to the DFT coefficients of the top row [PAP `77] in reversed order. These eigenvalues are:

$$\eta(k) = \sum_{n=0}^{M-1} c(n) W^{-nk}$$

where $W=e^{j2\pi k/M}$ and $C(e^{j\omega})$ represents the channel frequency response. Thus η(k) are obtained by sampling uniformly $C(e^{j\omega})$ at M frequency. For L<n<M c(n)=0. The circulant matrix can be diagonalized with DFT matrix:

$$\mathbf{C = W^{-1}\Lambda_C W}$$

where

$$\Lambda_C = \text{diag}\{C_M[0]\ldots C[M-1]\}$$

and $C_M[k]= \sum_{n=0}^{L} C(n) W^{nk}$ =M – point DFT of c(n)

is a permuted version of the eigenvalues η(k). The implementation of the communication system can be represented as shown in fig. 5.

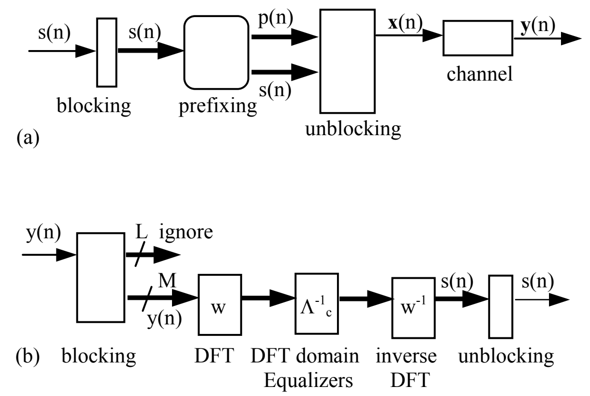

Figure 5. Block diagram description of the system based on cyclic prefix. (a) Transmitter, and (b) receiver

The diagonal elements of $\mathbf{\Lambda_c^{-1}}$ are 1/ $C_M(k)$ and can be regarded as a set of DFT-domain equalizers. All the complexity is done at the receiver, but in fig. 6 the DFT is done at the transmitter as in DMT system. If the channel is known it can move also $\mathbf{\Lambda_c^{-1}}$ and **W** to the transmitter part, yielding a useful configuration for cases where the receiver has to be the simplest.

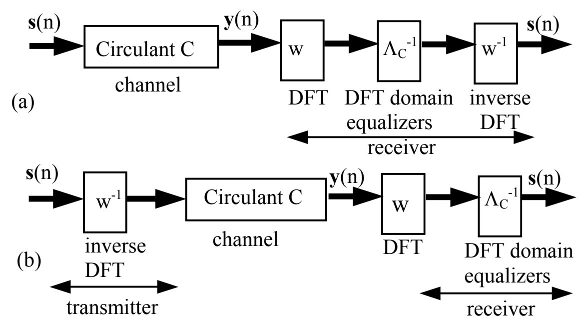

Figure 6. (a) A simplified schematic of the cyclic prefix system, and (b) practically useful rearrangement similar to conventional DMT system

If the channel is non-minimum phase having no zeros on the unit circle, **C** is non-singular and it can invert (4) to obtain the input symbol stream. In the cyclic prefix method, the DFT coefficients are bounded $|C_M[k]|\geq 1$ so that the diagonal elements 1/ $C_M[k]$ of the equalizer $\mathbf{\Lambda_c^{-1}}$ do not amplify noise.
When the channel is unknown, the fig. 6 is modified into fig. 7. The **r(n)** vector is a known vector of L+1 nonzero samples, N=M+L+1, and **s(n)** is a vector of M samples in nth block. Then

$$\mathbf{t}(n) = \begin{bmatrix} \mathbf{r}(n) \\ \mathbf{s}(n) \end{bmatrix}$$

In fig. 7 the subscript N indicates the size of the matrices and distinguishes them from fig. 6.

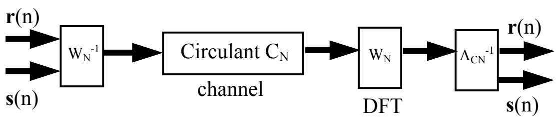

Fig. 7. The cyclic prefix system for blind identification

The C_N is a N x N circulant, $\mathbf{W_N}$ is N x N DFT matrix, and diagonal elements of Λ_{cN}^{-1} are $1/C_N[k]$ where:

$$C_N[k]=\sum_{n=0}^{L} c(n)e^{j2\pi nk/N}$$

Since the channel is unknown we cannot insert the matrix Λ_{cN}^{-1} yet, but it can measure the signal v(n), which has the top L+1 component scaled by $C_N[0]$, $C_N[1]$, …, $C_N[L]$. Since M(n) has known nonzero samples, it can therefore identify $C_N[0]$, $C_N[1]$, …, $C_N[L]$, related to the channel c[0], c[1], …, c[L] by the top/left (L+1)x(L+1) submatrix $\mathbf{W_{sub}}$ of the N x N DFT matrix. The channel can so be identified. Because in practice **v(n)** has an additive noise component from the channel, it can average the estimated $C_N[k]$ over a large number of the output block, the compromise being the latency involved. Also for the channel that vary in time, the rate at which it can update the blind estimation is compromised [VV-3]. The blind identification is possible only at the expense of the additional redundancy of L+1 samples, so the complete system has a higher bandwidth expansion factor (M+2L+1)/M. As in other methods this approaches unity as M grows.

III. TRANSCEIVER SYSTEM AND FADING CHANNEL MODEL

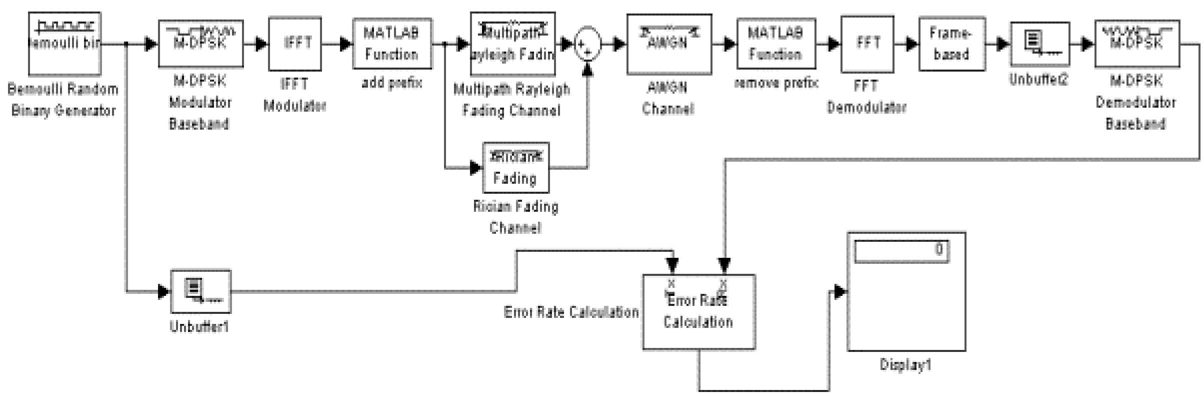

Fig. 8: Block diagram of the OFDM-MDPSK system

The block diagram of OFDM-MDPSK system is shown in (8). The serial information data are first grouped into blocks of M symbols. Then the symbols are parallelized by the S/P converter to a_{ki} and then applied to a differential encoder to generate the MDPSK signals. The transmitted signal can be expressed in complex form as:

$$s(t)=\sum_{i=-\infty}^{\infty} g(t-iT_s)\sum_{k=0}^{M-1} c_{ki}e^{j2\pi f_k(t-iT_S)} \qquad (5)$$

where $f_k=f_c+kT_s$, is the frequency of the k-th carrier and f_c is the lowest carrier frequency. $T_s=\Delta+t_s$ is the block duration where Δ and t_s are the circular prefix and the observation period, respectively. $c_{ki}=c_{k(i-1)}a_{ki}$ is chosen out of the constellation set $\left\{ e^{j2\pi m/M} \middle| m=0,1,...M-1 \right\}$, is an output of the k-th differential encoder in the time interval $[iT_s-\Delta,\ iT_s+t_s]$. g(t) is a pulse waveform of each symbol defined as (5):

$$g(t)=\begin{cases} 1 & for -\Delta \le t \le t_s \\ 0 & otherwise \end{cases}$$

The transmitted signal s(t) is subjected to multipath fading and AWGN, so the received signal can be written as (5)

$$r(t)=\int_{0}^{\infty} s(t-\tau)h(t,\tau)d\tau + n(t) \qquad (6)$$

where n(t) is a complex gaussian noise and h(t,τ) is the impulse response of the multipath fading channel at time t-τ. The frequency selective Rician fading channel here is modeled as a 3-ray tapped delay line with one line of sight (LOS) path and two multipath components.

$$h(t,\tau) = \sqrt{2P_s}\,\delta(t) + \sqrt{P_1}\,h_1(t)\delta(\tau-\tau_1) + \\ + \sqrt{P_2}\,h_2(t)\delta(\tau-\tau_2) \qquad (7)$$

where P_s is the power of LOS signals, P_1 and P_2 are the powers of the multipath signals. τ_1 and τ_2 are the delay of the first and second multipath respectively, and $0<\tau_1<\Delta<\tau_2$. h1(t) and h2(t) are independent slowly varying complex Gaussian random process with maximum Doppler shift f_m and are normalized as:

$$E[|h_1(t)|^2] = E[|h_2(t)|^2] = 2 \qquad (8)$$

$$E[|h_1(t)h_1^*(t+T_s)|] = E[|h_2(t)h_2^*(t+T_s)|] = \\ = 2J_0(2\pi f_m T_s) \qquad (9)$$

The multitone duration, T_s, is much longer than the serial symbol duration, T, so that the Doppler shift should not be ignored in (9).

A parameter characterizing the nature of the Rician fading channel is the Rice factor defined as the ratio of LOS component power P_s and the multipath components power $P_d= P_1+ P_2$, i.e. $K=P_s/P_d$. As special case the channel is AWGN channel (no multipath components) when $K\to\infty$, and a Rayleigh fading channel (no LOS component) when K=0. The received signal-to-noise ratio is defined as the average received bit energy divided by N_0 which is one sided power spectral density of AWGN in Watts/Hz, which is expressed as

$$\gamma_b = \frac{E_b}{N_0} = \frac{(P_s+P_d)T_s}{N_0\log_2 M} = \frac{(P_s+P_d)}{B_s N_0 \log_2 M} \\ = \frac{K+1}{\log_2 M}\cdot\frac{P_d}{P_n} \qquad (10)$$

where $B_s=1/T_s$ and $P_n=B_sN_0$ is the power of the filtered AWGN.

At the receiver, after doing FFT to the received signal, the output of the m-th subchannel at time iT_s can be obtained as (5)

$$r_{l,mi} = \frac{1}{t_s}\int_{iT_s}^{t_s+iT_s}\{\int_0^\infty s(t-\tau)h(t,\tau)d\tau + n(t)\}\cdot e^{-j2\pi f_M(t-iT_s)}\cdot dt \qquad (11)$$

for the l-th branch (l=1,2,3,…,L).

Finally, the differential detector of each subchannel uses the variable $\sum_{l=1}^{L} z_{l,mi} = \sum_{l=1}^{L} r_{l,mi}r^*_{l,m(i-1)}$ to decide which symbol was transmitted and then the parallel data are converted back to serial data stream .

IV. MEASUREMENTS AND CONCLUSIONS

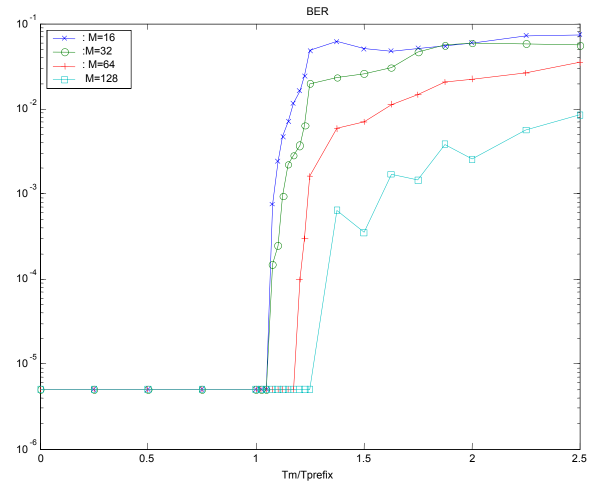

Fig.9: Bit error rate as a function of the maximum multipath delay, normalized with the cyclic prefix length, for a cyclic prefix duration equal with the duration of 4 serial symbols

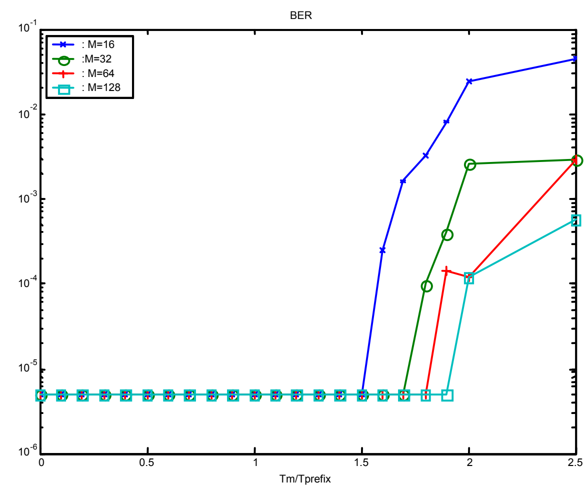

Fig.10: Bit error rate as a function of the maximum multipath delay , normalized with the cyclic prefix length, for a cyclic prefix duration equal with the duration of one serial symbol

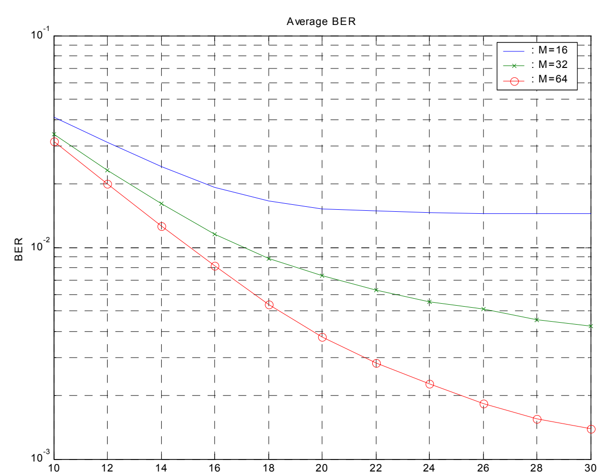

Fig. 11a: Average bit error rate for a two ray channel with one symbol period delay for different values of the frame lengths, M

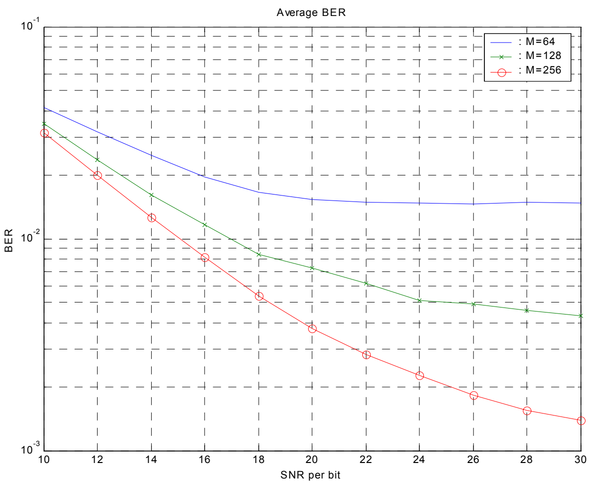

Fig. 11b: Average bit error rate for a two ray channel with four symbol periods delay and different frame lengths, M

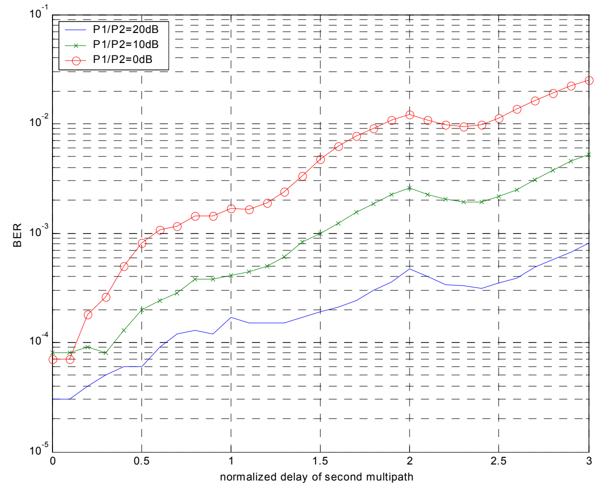

Fig 13: BER performances of a 3-ray Rician fading channel, MDPSK, E_b/N_0=40dB, M=32 and f_m*T_s=0.01

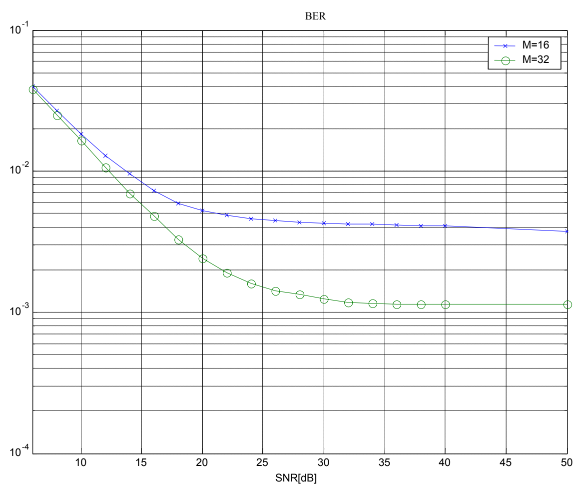

Fig. 12a: BER performances of a 3-ray Rician fading channel, MDPSK, $P_s/(P_1+P_2)$=10 dB, P_1/P_2=0dB, $\rho=\delta_2/T$=1.2 (M=16, 32)

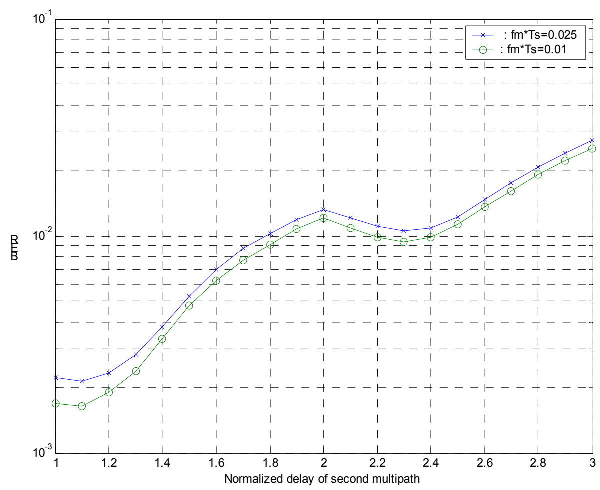

Fig. 14: BER performanes of a 3-ray Rician fading channel, MDPSK, E_b/No=40 dB, P1/P2=0dB and M=32

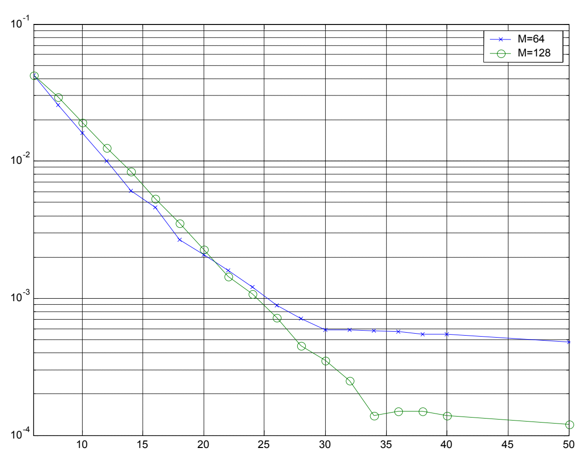

Fig. 12b: The same as 12a, but for other lengths of data blocks, M= 64, 128

In these simulations, it can be observed that the BER is improved by the length of the block M, fig. 9, 10, 11(a, b), 12(a,b) [LON'02]. The performance can significantly be improved when the number of subchannel is large because the rate of each subchannel decreases, so the results are according with the Shannon's theorem for the noisy channels. The BER performance is also improved by the length of the cyclic prefix (fig. 9,10) but the increase of the cyclic prefix length is payed by the increase of the bandwidth.

Fig. 11a shows the simulation results for a two-ray channel when the delay between the two rays is one symbol period. To mentain the same BER when the delay between the two rays is four symbol periods, a four times longer frame length is required(fig. 11b).

In fig. 12 a, b is plotted the BER performance with respect to average SNR per bit for a Rice channel. It can be seen

that the performance is improved by increasing the block length.
Analyzing the fig. 13 we can see that the performance increases by the increase of the P_1/P_2 ratio because of decrease of ISI and ICI (Inter Channel Interference) power introduced by the second multipath component.
In fig. 14, it is shown that maximum Doppler shift has significant influence on the BER when the delay of the second multipath component is small. When the delay is large, the main interference comes from ISI and the effect of the Doppler shift decades.

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