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TRANSMULTIPLEXERS: PERFECT RECONSTRUCTION AND COMPENSATION OF CHANNEL DISTORTION*

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Abstract. This paper presents new results for a multi-input, multi-output transmultiplexer and describes the analogy with known results for a single-input, single-output subband system. First, the perfect reconstruction property in both systems is explored. Then, the complementary nature of the two systems is examined and is interpreted both in terms of network duality and by a series of block diagram manipulations which convert a subband system to a transmultiplexer. It is also shown that an interchange of the set of combining filters and separation filters preserves the crosstalk-free nature of transmultiplexers. The problem of channel distortion is alleviated by passing the received composite signal through a channel compensation filter or equalizer. Five methods for specifying the compensation filter are proposed, each of which reinstates the crosstalk-free nature of the transmultiplexer. However, residual intersymbol interference remains. Two of the five approaches attempt to suppress the intersymbol interference. A comparison of the performance of the five methods is done for a particular channel.

Zusammenfassung. Neue Ergebnisse für einen Transmultiplexer mit mehreren Ein- und Ausgängen werden präsentiert. Die Analogie zu bekannten Resultaten für Teilbandsysteme mit einem Eingang und einem Ausgang wird beschrieben. Zunächst wird untersucht, wie in beiden Systemen eine fahlerfreie Rekonstruktion gelingt. Dann wird die komplementäre Natur beider Systeme untersucht und interpretiert sowohl im Sinne der Netzwerk-Dualität als auch durch eine Reihe von Manipulationen im Blockdiagramm, die aus einem Teilbandsystem einen Transmultiplexer machen. Es wird auch gezeigt, daß ein Vertauschen der Synthesefilter mit den Analysefiltern die Transmultiplexer-Eigenschaft der Nebensprechfreiheit nicht beeinträchtigt. Das Problem der Kanalverzerrungen wird gemildert, indem das empfangene zusammengesetzte Signal durch ein Kanal-Kompensationsfilter oder einen Entzerrer geschickt wird. Fünf Verfahren zur Spezifikation des Kompensationsfilters werden vorgeschlagen; sie alle erhalten die nebensprechfreiheit des Transmultiplexers. Eine gewisse Intersymbol-Interferenz ist jedoch unvermeidlich. Zwei der fünf Verfahren stellen den Versuch dar, die Intersymbol-Interferenz zu unterdrücken. Die fünf Ansätze werden im Falle eines speziellen Kanals miteinander verglichen.

Résumé. De nouveaux développements au sujet de transmultiplexeurs à entrées et sorties multiples sont présentés dans cet article ainsi qu'une description de l'analogie entre ces résultats et ceux obtenus pour des systèmes en sous-bandes à entrées et sorties uniques. En premier lieu, la propriété de reconstruction parfaite de chaque système est examinée. Par la suite, la dualité entre les deux systèmes est examinée et interprétée par une série de manipulations de schéma blocs qui ont pour effet de convertir un système en sous-bandes en un transmultiplexeur. Il est aussi démontré que l'opération sans diaphonie des transmultiplexeurs est préservée lorsqu l'ensemble des filtres de reconstruction et des filtres de séparation sont interchangés. Le problème de distorsions introduites par le canal est évité en filtrant le signal de réception avec un filtre de compensation ou un égalisateur. Cinq méthodes pour réaliser l'égalisateur sont proposées. Chacune de ces méthodes élimine complètement la diaphonie précédemment introduite par le système. Par contre, l'interférence entre symboles n'est pas éliminée. Deux des cinq méthodes proposées tentent de supprimer cette interférence. Les cinq méthodes sont comparées pour un canal de transmission particulier.

Keywords. Transmultiplexers, subband systems, perfect reconstruction, channel distortion, channel compensation filter.

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1. Introduction

Multirate filter banks can be used to realize subband systems and transmultiplexers [2, 12, 14]. A subband system as depicted in Fig. 1 finds applications in speech coding. It is a single-input, single-output system in which the analysis filters split the input signal spectrum into frequency bands which generally overlap. The resultant signals are then decimated. These decimated signals in each band contain aliased components of the input signal. The interpolation step followed by the parallel action of the synthesis filters serve to cancel the aliased components thereby restoring the original signal. The literature to date has discussed the issues regarding the design of the analysis and synthesis filters [1, 3, 5, 6, 9, 11] and the computational complexity involved in implementing multirate systems [1, 8].

A transmultiplexer as depicted in Fig. 2 is a multi-input, multi-output system that finds applications in data transmission. In particular, transmultiplexers are useful in achieving simultaneous transmission of many data signals across a single channel. The multiplexing is performed by interpolating and filtering different input signals into one composite signal. The composite signal is passed through a parallel structure of *separation* filters and decimators to yield the output signals. The *combining* filters serve to allocate different portions of the channel bandwidth to the various input signals. In general, the system has spectral overlap which leads to crosstalk. Crosstalk exists when an output signal is influenced by signals from bands other than its corresponding input. Another type of distortion, namely intersymbol interference, arises when a sample of the output signal is dependent on past and future input samples and not on its corresponding input sample alone. The separation filters can be chosen to cancel both crosstalk and intersymbol interference thereby allowing for the perfect reconstruction of each of the input data signals. Vetterli [12, 14] has explored some properties related to the complementary nature of subband systems and transmultiplexers.

This paper focusses on the perfect reconstruction property and the effects of channel distortion in transmultiplexers. In typical applications of transmultiplexers (data transmission) and subband systems (speech coding), the perfect reconstruction property is required for restoration of the original signal(s). Previously, methods to configure the filter banks for achieving perfect reconstruction in transmultiplexers and subband systems have been proposed in [11, 12, 14]. Also, the effects of channel distortion for subband systems are studied in [10]. In this paper, analogous results for transmultiplexers are developed. In Section 2, we discuss the perfect reconstruction property for subband systems and transmultiplexers and point out the differences between the two systems. The complementary nature of the two systems is described by both a block diagram interpretation and a network theoretic approach. Section 3 demon-

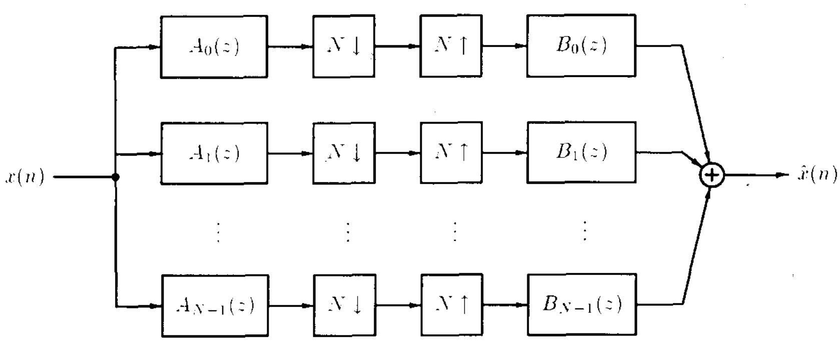


Fig. 1. A subband system.

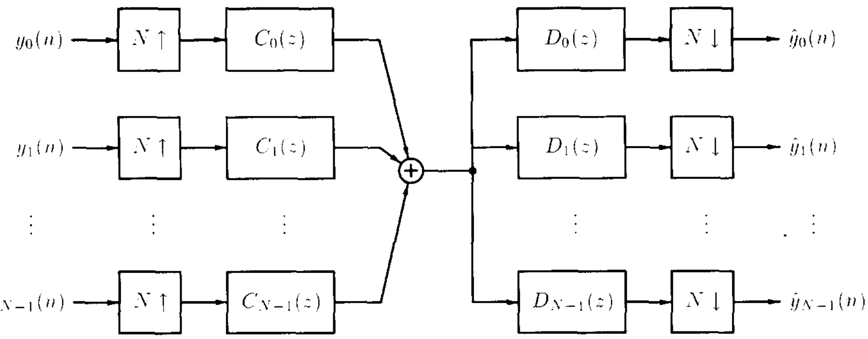


Fig. 2. A transmultiplexer system.

strates how to combat channel distortion effects in a transmultiplexer based on a result derived by a matrix formalism. A performance evaluation is done for specific channels.

2. Perfect reconstruction and network duality

This section first gives appropriate background material on subband systems and transmultiplexers. We introduce a block diagram interpretation of the complementary nature of the two systems. The application of delay factors in both systems illustrate some of the differences in achieving perfect reconstruction. Finally, we prove the swapping property for transmultiplexers (previously shown for subband systems) and provide a network theoretic interpretation of this property for both systems.

2.1. Subband system

In the subband system, the general form of the relationship between the input $X(z)$ and the output $\hat{X}(z)$ is

$$\hat{X}(z)=\frac{1}{N}\sum_{l=0}^{N-1} X(zW^{-l})T_l(z), \tag{1}$$

where the aliasing functions $T_l(z)$ are given by

$$T_l(z)=\sum_{k=0}^{N-1} A_k(zW^{-l})B_k(z) \tag{2}$$

and $W=\exp(-j2\pi/N)$. Aliasing due to frequency shifted versions of $X(z)$ is cancelled if and only if

$$\sum_{k=0}^{N-1} A_k(zW^{-l})B_k(z)=0 \quad \text{for} \quad 1\leqslant l\leqslant N-1. \tag{3}$$

After cancellation of the aliasing, the input-output transfer function $T(z)$ is

$$T(z)=\sum_{k=0}^{N-1} A_k(z)B_k(z). \tag{4}$$

Then $\hat{X}(z)=T(z)X(z)/N$. In addition, if $T(z)=cz^{-p}$, perfect reconstruction is accomplished in that there is neither amplitude nor phase distortion. The output samples are merely scaled and delayed versions of the input samples.

2.2. Transmultiplexer

An N band multi-input, multi-output transmultiplexer generates input-output relationships given by

$$\hat{Y}_i(z)=\frac{1}{N}\sum_{k=0}^{N-1} Y_k(z) \times \sum_{l=0}^{N-1} C_k(z^{1/N}W^{-l})D_i(z^{1/N}W^{-l}) \quad \text{for} \quad 0\leqslant i\leqslant N-1, \tag{5}$$

or equivalently in matrix form (note the change from z to z^N),

$$\hat{\boldsymbol{Y}}^{\mathrm{T}}(z^N)=\frac{1}{N}\boldsymbol{Y}^{\mathrm{T}}(z^N)\boldsymbol{C}(z)\boldsymbol{D}^{\mathrm{T}}(z) = \frac{1}{N}\boldsymbol{Y}^{\mathrm{T}}(z^N)\boldsymbol{T}(z^N), \tag{6}$$

where

$$\boldsymbol{C}(z)=\begin{bmatrix} C_0(z) & C_0(zW^{-1}) & \cdots & C_0(zW^{-N+1}) \\ C_1(z) & C_1(zW^{-1}) & \cdots & C_1(zW^{-N+1}) \\ \vdots & \vdots & & \vdots \\ C_{N-1}(z) & C_{N-1}(zW^{-1}) & \cdots & C_{N-1}(zW^{-N+1}) \end{bmatrix}, \tag{7}$$

$$\boldsymbol{D}(z)=\begin{bmatrix} D_0(z) & D_0(zW^{-1}) & \cdots & D_0(zW^{-N+1}) \\ D_1(z) & D_1(zW^{-1}) & \cdots & D_1(zW^{-N+1}) \\ \vdots & \vdots & & \vdots \\ D_{N-1}(z) & D_{N-1}(zW^{-1}) & \cdots & D_{N-1}(zW^{-N+1}) \end{bmatrix}, \tag{8}$$

$$\hat{\boldsymbol{Y}}(z^N)=\begin{bmatrix} \hat{Y}_0(z^N) \\ \hat{Y}_1(z^N) \\ \vdots \\ \hat{Y}_{N-1}(z^N) \end{bmatrix} \tag{9}$$

and

$$\boldsymbol{Y}(z^N)=\begin{bmatrix} Y_0(z^N) \\ Y_1(z^N) \\ \vdots \\ Y_{N-1}(z^N) \end{bmatrix}. \tag{10}$$

The output signals $\hat{Y}_i(z^N)$ are related to the input signals $Y_k(z^N)$ via a transfer function matrix $\boldsymbol{T}(z^N)$ whose elements are

$$T_{ki}(z^N)=\sum_{l=0}^{N-1} C_k(zW^{-l})D_i(zW^{-l}). \tag{11}$$

The objective is to eliminate crosstalk such that each output signal $\hat{Y}_i(z)$ depends only on its corresponding input $Y_i(z)$ and not on other input signals. Hence, it is desired that the crosstalk transfer functions $T_{ki}(z^N)=0$ for $k\neq i$. Furthermore, if each of the transfer functions $T_{kk}(z^N)=T(z^N)$, the relationship between each of the inputs and their corresponding outputs is identical. For an input-output transfer function $T(z^N)$, intersymbol interference may be present. Intersymbol interference is eliminated if $T(z)=cz^{-p}$. Then, perfect reconstruction to within a delay and a gain constant is achieved.

From the above development, it is seen that the elimination of crosstalk and the achievement of identical input-output relations $T(z^N)$ are equivalent to configuring the combining and separation filters to satisfy the matrix equation

$$\boldsymbol{C}(z)\boldsymbol{D}^{\mathrm{T}}(z)=T(z^N)\boldsymbol{I}, \tag{12}$$

where $\boldsymbol{I}$ is the identity matrix. If the above matrix equation is satisfied, each of the output signals $\hat{Y}_k(z)=T(z)Y_k(z)/N$.

2.3. Network duality and complementary systems

The subband system of Fig. 1 consists of the cascade connection of two subsystems. One is a single-input, multi-output subsystem consisting of the analysis bank and the decimators. The other subsystem comprises the interpolators and the synthesis bank. These two subsystems are duals of one another as can be seen by invoking the concept of network transposition [2]. In fact, the reverse cascade connection of the same two dual subsystems results in a transmultiplexer. Subband systems and transmultiplexers can be viewed as being complementary multirate structures.

The fundamental complementary nature between subband systems and transmultiplexers relates aliasing cancellation in the former to crosstalk elimination in the latter [14]. Aliasing cancellation in a subband system occurs if and only if

$$\boldsymbol{A}^{\mathrm{T}}(z)\boldsymbol{B}(z)=\operatorname{diag}[T(z), T(zW^{-1}), \ldots, T(zW^{-N+1})], \tag{13}$$

where $\boldsymbol{A}(z)$ and $\boldsymbol{B}(z)$ are defined just as $\boldsymbol{C}(z)$ and $\boldsymbol{D}(z)$ in (7) and (8). By relating the above equation to (12), it can be shown that any combining/separation filter banks that eliminate crosstalk and achieve the same input-output transfer function for all pairs of corresponding terminals in a transmultiplexer will cancel aliasing when utilized as analysis/synthesis filter banks in a subband system. However, the reverse is not true unless the input-output transfer function of the subband system is a function of z^N. Analysis/synthesis filter banks for a subband system that cancel aliasing and achieve an input-output transfer function in z^N result in the relationship $\hat{X}(z) = (1/N)T(z^N)X(z)$. These same filter banks eliminate crosstalk in a transmultiplexer and achieve $\hat{Y}_i(z) = (1/N)T(z)Y_i(z)$ for $i = 0$ to $N-1$.

A further interpretation of this result is as follows. Suppose we design a subband system that achieves perfect reconstruction. In general, this filter bank will not cancel crosstalk in a transmultiplexer unless the transfer function $T(z) = cz^{-p}$ has p which is a multiple of N. First suppose $c = N$ and $p = 0$. The resulting filter bank can be applied in either a subband system or a transmultiplexer. Furthermore, there is an exact complementary relationship since the two systems are identity systems (the output samples are identical to the corresponding input samples; there is no delay factor). This is further motivated from the sequence of block diagram manipulations shown in Fig. 3. The identity subband system allows us to connect the input and output and break the connections between the decimators and interpolators thereby forming an equivalent transmultiplexer that is also an identity system. Note that the analysis filter bank $\boldsymbol{A}(z)$ in a subband system corresponds naturally to the separation filter bank $\boldsymbol{D}(z)$ in a transmultiplexer. Also, there exists a similar correspondence between $\boldsymbol{B}(z)$ and $\boldsymbol{C}(z)$.

2.4. Delay factors

The identity subband system ($T(z) = N$) can be modified by adding delay factors to the analysis

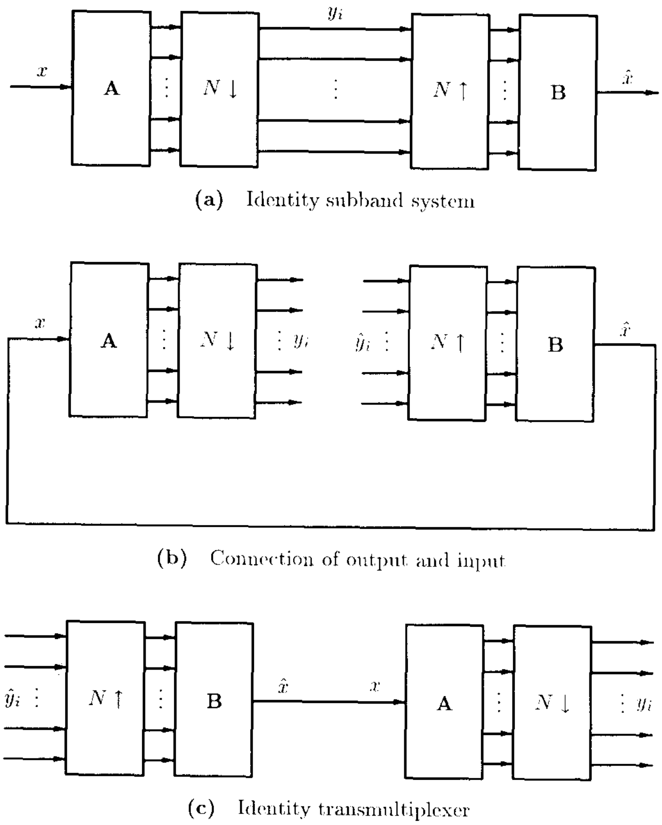


Fig. 3. Duality via block diagram manipulation.

and/or synthesis filter banks (see Fig. 4(a)). The same delay factor z^{-p_1} is applied to each analysis filter. Similarly, the delay factor z^{-p_2} is applied to each synthesis filter. Now, the input-output transfer function is $T(z) = Nz^{-p}$ where $p = p_1 + p_2$. In a practical approach, p_1 and p_2 are chosen so that a causal system results for both the analysis and synthesis filter banks.

Consider the application of delay factors to an identity transmultiplexer (see Fig. 4(b)). The same delay factor z^{-q_1} is applied to each combining filter. Similarly, the delay factor z^{-q_2} is applied to each separation filter. The constraint that $q_1 + q_2$ is equal to a multiple of N is necessary for crosstalk cancellation to be preserved. Otherwise, the decimators and interpolators operate out of phase and crosstalk will no longer be cancelled. In addition, if $q_1 + q_2$ is a multiple of N, the delays can be moved across the decimators and interpolators without disturbing the crosstalk-free nature of the system. With the delay factors, the input-output transfer function is $T(z^N) = Nz^{-(q_1+q_2)}$. The above discussion on the complementary systems can be made rigorous with the introduction of a matrix formalism. For instance, the delay factors applied in the transmultiplexer can be viewed as a channel filter to determine conditions under which perfect reconstruction is preserved (see Section 3).

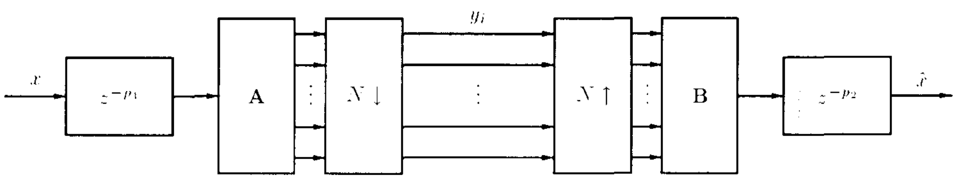
Lossless banks

As an example of the application of delay factors, consider lossless digital transfer functions which have been used in subband systems [11]. A matrix function $\boldsymbol{G}(z)$ is said to be *lossless* [11] if it is stable and satisfies the relation

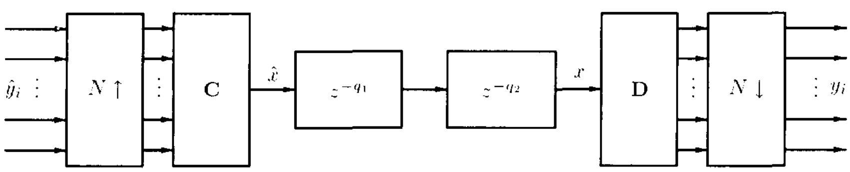
$$\boldsymbol{G}^{\mathrm{H}}(z^{-1})\boldsymbol{G}(z) = \boldsymbol{I}, \tag{14}$$

where the superscript H denotes the complex conjugation of the coefficients of each entry of the matrix followed by transposition. For a subband system, the analysis filters can be designed as cascaded FIR lattice structures such that the analysis filter matrix is lossless [11]. In order to obtain a perfect reconstruction subband system with $T(z) = cz^{-p}$, the synthesis filters are related to the analysis filters by $B_k(z) = cz^{-p}A_k(z^{-1})$. The factors $c = N$ and $p = 0$ result in an identity system.

Consider the use of a lossless combining filter bank in a transmultiplexer. Crosstalk is eliminated in a transmultiplexer if (12) is satisfied. Suppose $\boldsymbol{C}(z)$ is lossless. Then, premultiplying both sides of (12) by $\boldsymbol{C}^{\mathrm{H}}(z^{-1})$ gives $\boldsymbol{D}^{\mathrm{T}}(z) = T(z^N)\boldsymbol{C}^{\mathrm{H}}(z^{-1})$. By comparing the individual elements of this matrix equation, one can see that the separation filter bank is given by $D_k(z) = T(z^N)C_k(z^{-1})$. Moreover, if $T(z^N) = cz^{-mN}$, then both crosstalk and intersymbol interference are eliminated and



(a) Subband system



(b) Transmultiplexer

Fig. 4. Application of delay factors.

the magnitude responses of $C_k(z)$ and $D_k(z)$ are identical. An identity system emerges if $T(z^N) = N$. The procedure for the design of the combining filters with the purpose of satisfying the lossless property as given in [11] can be used for the transmultiplexer.

If the filter bank is designed to satisfy the lossless property using the approach in [11], then each filter is FIR. Consider FIR lossless banks $\boldsymbol{C}(z)$ and $\boldsymbol{A}(z)$ with $C_k(z) = A_k(z)$. Given identity subband systems and transmultiplexers, the filters $B_k(z)$ and $D_k(z)$ will be noncausal. We wish to cascade the synthesis and separation banks with delay factors to ensure causality and hence, have an input-output transfer function equal to a pure delay. For a subband system, any delay factor that permits $B_k(z)$ to be causal can be chosen. However, for the transmultiplexer, the delay element that forces the separation filter bank to be causal is constrained to be a multiple of N which is in general greater than or equal to the delay factor used for the subband system. There is less freedom in choosing the delay factor for the transmultiplexer.

2.5. Swapping property

Note that the mathematics dealing with the equivalence of aliasing and crosstalk cancellation only places restrictions on the matrix product $\boldsymbol{A}^{\mathrm{T}}(z)\boldsymbol{B}(z)$ or $\boldsymbol{C}(z)\boldsymbol{D}^{\mathrm{T}}(z)$ and not on the individual filters themselves. This motivates the swapping property discussed in this section. If $\boldsymbol{A}(z)$, $\boldsymbol{B}(z)$, $\boldsymbol{C}(z)$ or $\boldsymbol{D}(z)$ is singular, the appropriate matrix product cannot be made proportional to the identity matrix. In particular, this condition implies that no two filters in a filter bank can be identical, or more generally no filters can be a linear combination of other filters.

The swapping property addresses the question of whether or not exchanging the positions of the filter banks preserves the reconstruction property. Given an alias-free subband system, it has been shown [10] that swapping each analysis filter $A_k(z)$ with the corresponding synthesis filter $B_k(z)$ still results in an alias-free system. It is now shown that the same property holds for a transmultiplexer.

The input-output description of a transmultiplexer is given by (12). If the combining and separation filter banks are swapped, the new matrix product $\boldsymbol{D}(z)\boldsymbol{C}^{\mathrm{T}}(z) = \boldsymbol{T}^{\mathrm{T}}(z^N)$ that describes the input-output behaviour is equal to the old matrix product if $\boldsymbol{T}(z^N)$ is symmetric. This requires the crosstalk (or off-diagonal) terms to be symmetric. This occurs if $C_k(z) = D_k(z)$ but can also occur for more general cases. In particular, if the crosstalk is initially zero, swapping preserves the crosstalk-free nature and maintains the same input-output transfer functions. As in [10], the above derivation makes no specific assumptions about the filters or about N.

A more natural interpretation of swapping emerges from network transposition. Subband systems and transmultiplexers are configured by cascading two dual sybsystems each consisting of a filter bank as discussed earlier. By performing network transposition, we see that the duals of subband systems and transmultiplexers are again subband systems and transmultiplexers with the filter banks interchanged. Consider a subband system which is in general linear and time-varying. The dual subband system is also linear and time-varying but is described by different aliasing functions than the original system. A frequency shifted version of the aliasing function $T_l(z)$, namely, $T_l(zW^l)$, of the original system is equal to the aliasing function $T_{N-l}(z)$ of the dual system. The subband system becomes time-invariant when aliasing is cancelled and is described by an input-output transfer function $T(z)$. Therefore, the dual will also be alias-free and have the same $T(z)$ [2]. Therefore, as shown in [10], swapping preserves aliasing cancellation and maintains the same input-output transfer function.

Now, consider a transmultiplexer which in general is not crosstalk-free. The dual transmultiplexer is also not crosstalk-free. The input-output transfer functions $T_{kk}(z)$ ($k = 0$ to $N-1$) are the same for both systems. The crosstalk functions $T_{kl}(z)$ in the original network (relating the output at terminal l

to the input at terminal k) are equal to the functions $T_{lk}(z)$ of the dual network (relating the output at k to the input at l). If a transmultiplexer is crosstalk-free, swapping the filter banks results in a dual transmultiplexer that is also crosstalk-free and which has the same input-output transfer functions as the original system.

3. Combatting channel distortion

Channel distortion is introduced in a subband system when each of the intermediate decimated signals is passed through a linear channel. Given that the original system with no channel distortion eliminates aliasing, the procedure given in [10] describes how to modify the synthesis filters to combat channel distortion. Each of the synthesis filters is modified by a different factor that depends on the system function of each of the channels such that the cancellation of aliasing is reinstated. Note that even if the original system eliminates both amplitude and phase distortion, the suggested modification renders a new system in which both amplitude and phase distortion are present since the new input-output transfer function depends on the system function of the channels. The channel system functions are often not under the control of the designer.

In a transmultiplexer, there is one composite signal that passes through a single channel. The channel is assumed to be linear and has a system function $Q(z)$. The overall system is shown in Fig. 5. Given that the combining and separation banks are configured to satisfy $\boldsymbol{C}(z)\boldsymbol{D}^{\mathrm{T}}(z) = T(z^N)\boldsymbol{I}$ (same input-output transfer function for every pair of terminals and the absence of crosstalk), several approaches are formulated to specify a channel compensation filter to again ensure the cancellation of crosstalk.

3.1. Theoretical development

The outputs $\hat{Y}_i(z)$ as in Fig. 5 are given by

$$\hat{\boldsymbol{Y}}^{\mathrm{T}}(z^N) = \frac{1}{N}\boldsymbol{Y}^{\mathrm{T}}(z^N)\boldsymbol{C}(z)\boldsymbol{Q}(z)\boldsymbol{D}^{\mathrm{T}}(z), \qquad (15)$$

where

$$\boldsymbol{Q}(z) = \mathrm{diag}[Q(z), Q(zW^{-1}), \ldots, Q(zW^{-N+1})]. \qquad (16)$$

Since the system with no channel distortion ($Q(z) = 1$) eliminates crosstalk, it satisfies (12). To cancel crosstalk with the presence of a channel, one needs to satisfy the augmented equation

$$\boldsymbol{C}(z)\boldsymbol{Q}(z)\boldsymbol{D}^{\mathrm{T}}(z) = S(z^N)T(z^N)\boldsymbol{I}. \qquad (17)$$

In the sequel, it is assumed that $Q(z)$ is a stable function. No restriction on the zeros of $Q(z)$ is imposed. In combatting channel distortion, the idea is to place a channel compensation filter or equalizer $E(z)$ in cascade with the channel $Q(z)$ to cancel crosstalk. This is equivalent to modifying the separation filters to $D'_k(z) = D_k(z)E(z)$. Then, a new separation filter matrix $\boldsymbol{D}'(z) = \boldsymbol{D}(z)\boldsymbol{R}(z)$ results where

$$\boldsymbol{R}(z) = \mathrm{Diag}[E(z), E(zW^{-1}), \ldots, E(zW^{-N+1})]. \qquad (18)$$

If $\boldsymbol{R}(z)$ is chosen such that $\boldsymbol{Q}(z)\boldsymbol{R}(z) = S(z^N)\boldsymbol{I}$, (17) becomes

$$\begin{aligned}\boldsymbol{C}(z)\boldsymbol{Q}(z)[\boldsymbol{D}'(z)]^{\mathrm{T}} &= \boldsymbol{C}(z)\boldsymbol{Q}(z)\boldsymbol{R}(z)\boldsymbol{D}^{\mathrm{T}}(z)\\ &= S(z^N)\boldsymbol{I}\boldsymbol{C}(z)\boldsymbol{D}^{\mathrm{T}}(z)\\ &= S(z^N)T(z^N)\boldsymbol{I}. \qquad (19)\end{aligned}$$

In choosing $\boldsymbol{R}(z)$, the stability of $E(z)$ must be ensured.

The special case in which the channel response $Q(z)$ is itself a function of z^N ensures that $\boldsymbol{C}(z)\boldsymbol{Q}(z)\boldsymbol{D}^{\mathrm{T}}(z)$ remains a function of z^N and, consequently, that no crosstalk is introduced by the channel [13]. A particular case is when $Q(z)$ is a pure delay of the form z^{-mN}. This was dealt with in Section 2 when delay factors were applied to an identity transmultiplexer. When these delay factors are viewed as a channel, the above matrix manipulation confirms that the transmultiplexer remains crosstalk-free. In addition, the perfect reconstruction property is preserved for an identity transmultiplexer.

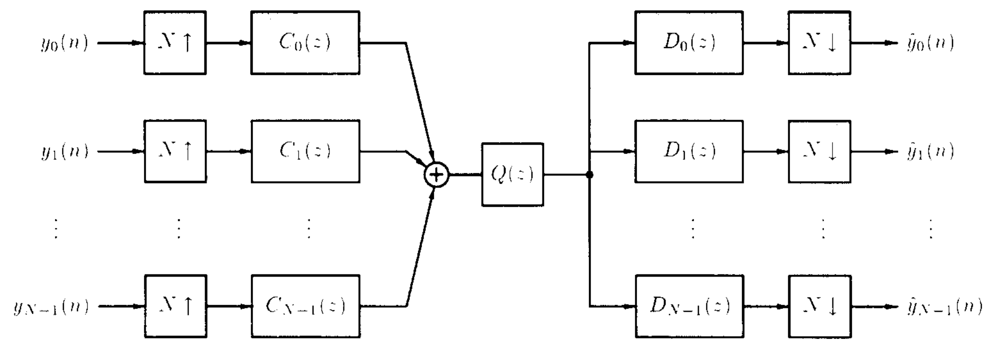


Fig. 5. Channel distortion in a transmultiplexer.

An obvious solution to $\boldsymbol{Q}(z)\boldsymbol{R}(z)=S(z^N)\boldsymbol{I}$ is to choose $\boldsymbol{R}(z)=\boldsymbol{Q}^{-1}(z)$. This makes $S(z^N)=1$ and $E(z)=1/Q(z)$. However, this solution is inappropriate if $Q(z)$ has zeros outside the unit circle since an unstable compensation filter $E(z)$ results.

To achieve crosstalk cancellation, $\boldsymbol{R}(z)$ is set to be

$$\boldsymbol{R}(z)=V(z^N)\boldsymbol{I}\boldsymbol{C}_Q(z), \tag{20}$$

where $V(z^N)$ is any arbitrary function of z^N and $\boldsymbol{C}_Q(z)$ is the cofactor matrix of $Q(z)$ given by

$$\boldsymbol{C}_Q(z)=\begin{bmatrix} \prod_{\substack{i=0\\ i\neq 0}}^{N-1} Q(zW^{-i}) & 0 & \cdots & 0 \\ 0 & \prod_{\substack{i=0\\ i\neq 1}}^{N-1} Q(zW^{-i}) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \prod_{\substack{i=0\\ i\neq N-1}}^{N-1} Q(zW^{-i}) \end{bmatrix} \tag{21}$$

Then

$$\boldsymbol{Q}(z)\boldsymbol{R}(z)=V(z^N)\prod_{i=0}^{N-1} Q(zW^{-i})\boldsymbol{I} \tag{22}$$
$$=S(z^N)\boldsymbol{I}$$

and

$$E(z)=V(z^N)\prod_{i=1}^{N-1} Q(zW^{-i}). \tag{23}$$

The compensation filter $E(z)$ is actually composed of two filters. The filter with system function $\prod_{i=1}^{N-1} Q(zW^{-i})$ when cascaded with $Q(z)$ can be viewed as a composite channel $\prod_{i=0}^{N-1} Q(zW^{-i})$, which, being a function of z^N, ensures the cancellation of crosstalk. However, residual intersymbol interference remains. The other filter $V(z^N)$ should be a function of z^N to preserve the crosstalk cancellation property. However, its actual role is to control the residual intersymbol interference. In effect, the channel compensation filter consists of two components, one which exactly cancels crosstalk and one which controls intersymbol interference. Since $Q(z)$ is stable, it follows that $E(z)$ is stable providing $V(z^N)$ is stable. Based on the specification for $\boldsymbol{R}(z)$, different approaches of choosing $V(z^N)$ are given.

3.1.1. Choices for $V(z^N)$

METHOD 1. The simplest method, namely, $V(z^N)=1$ does not attempt to control the intersymbol interference. It introduces the factor $S(z^N)=\prod_{i=0}^{N-1} Q(zW^{-i})$ in the overall input-output transfer function.

METHOD 2. A second procedure which is similar to the one outlined in [10], alleviates the problem of a high order input-output transfer function that is present in the previous approach. Suppose $Q(z)$ is split up as

$$Q(z)=\frac{N(z)}{D(z)}=\frac{U_+(z)U_-(z)}{D(z)}, \tag{24}$$

where $U_+(z)$ contains the zeros of $Q(z)$ within the unit circle and $U_-(z)$ contains the zeros of $Q(z)$ on and outside the unit circle. Since $Q(z)$ is assumed to be stable, $D(z)$ has all its zeros within the unit circle. By setting

$$V(z^N) = \prod_{i=0}^{N-1} \frac{D(zW^{-i})}{U_+(zW^{-i})}, \tag{25}$$

we get a lower order factor in the input-output transfer function

$$S(z^N) = \prod_{i=0}^{N-1} U_-(zW^{-i}) \tag{26}$$

and a stable channel compensation filter

$$E(z) = \left[\frac{D(z)}{U_+(z)} \prod_{i=1}^{N-1} U_-(zW^{-i})\right]. \tag{27}$$

Since the order of the input-output transfer function is reduced over that of Method 1, the resulting time span of the intersymbol interference is shortened.

METHOD 3. Assume that the original transfer function $T(z^N)$ is allpass and that $Q(z)$ has no zeros on the unit circle (in analogy with the development in [10]). Now, we proceed to examine whether the allpass property of the input-output transfer function can be preserved. Setting

$$V(z^N) = \prod_{i=0}^{N-1} \frac{D(zW^{-i})}{U_+(zW^{-i})U_-(z^{-1}W^{i})} \tag{28}$$

renders a new allpass factor

$$S(z^N) = \prod_{i=0}^{N-1} \frac{U_-(zW^{-i})}{U_-(z^{-1}W^{i})} \tag{29}$$

and a stable channel compensation filter

$$E(z) = \frac{D(z)}{U_+(z)} \frac{\prod_{i=1}^{N-1} U_-(zW^{-i})}{\prod_{i=0}^{N-1} U_-(z^{-1}W^{i})}. \tag{30}$$

Method 3 preserves the allpass property of the input-output transfer function but introduces an infinite time span for the intersymbol interference.

METHOD 4. So far, we have presented methods that either control the time span of the intersymbol interference or the allpass nature of the input-output transfer function. Now, we attempt to choose an FIR $V(z^N)$ so as to suppress the intersymbol interference. Given that

$$S(z^N) = V(z^N) \prod_{i=0}^{N-1} Q(zW^{-i}) = V(z^N)F(z^N), \tag{31}$$

or equivalently $S(z) = V(z)F(z)$, we determine the coefficients of an FIR $V(z)$ to minimize the mean-square intersymbol interference $\sum_{n\neq 0} s^2(n)$. Since $s(n) = v(n) * f(n)$, it can be shown that $\sum_{n\neq 0} s^2(n) = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{F}\boldsymbol{v}$, where $\boldsymbol{v}$ is the column vector of coefficients of $V(z)$ and $\boldsymbol{F}$ is a positive definite symmetric matrix whose entries $F(k, l)$ are given by

$$F(k, l) = \sum_{\substack{n \\ n\neq 0}} f(n-k)f(n-1). \tag{32}$$

To avoid the trivial solution $V(z) = 0$, we impose the constraint $\boldsymbol{v}^{\mathrm{T}}\boldsymbol{v} = 1$. Then, $\boldsymbol{v}$ is the eigenvector corresponding to the minimum eigenvalue of $\boldsymbol{F}$. Note that Method 4 can be viewed as attempting to approximate the inverse of the composite channel $\prod_{i=0}^{N-1} Q(zW^{-i})$.

METHOD 5. An alternative method to suppress the intersymbol interference is to choose $V(z^N)$ to be

$$V(z^N) = W(z^N)\left[\prod_{i=0}^{N-1} \frac{D(zW^{-i})}{U_+(zW^{-i})}\right]. \tag{33}$$

Then

$$S(z^N) = W(z^N) \prod_{i=0}^{N-1} U_-(zW^{-i}). \tag{34}$$

An FIR $W(z^N)$ is determined to suppress the mean-square intersymbol interference. As compared to Method 4, only an approximation of the inverse of a maximum phase function that contains the zeros of $Q(z)$ on and outside the unit circle is done. A factor of $V(z^N)$ exactly cancels the zeros and poles of $Q(z)$ within the unit circle.

SUMMARY OF METHODS. Table 1 shows the compensation filter $E(z)$ and the input-output transfer functions $T(z^N)S(z^N)$ resulting from the methods presented above. Suppose we have an FIR channel $Q(z)$. This leads to either an FIR or IIR compensation filter depending on the method utilized. Assuming that $T(z^N)$ is an FIR function (this is often the case since perfect reconstruction is desired), the input-output transfer function is FIR in methods 1, 2, 4 and 5. Method 3 is only useful for an allpass $T(z^N)$ and renders an IIR input-output transfer function.

An IIR channel results in IIR equalizers for all the methods. However, Methods 1, 3 and 4 produce an IIR input-output transfer function while Methods 2 and 5 still produce an FIR input-output transfer function (under the assumption that $T(z^N)$ is FIR). Methods 2, 3 and 5 involve additional computation to split the numerator of $Q(z)$ into its minimum and maximum phase parts.

For the special case when $Q(z)$ is a function of z^N, crosstalk is not introduced. Then, the compensation filters for Methods 1 and 4 assume a special form. Method 1 renders a compensation filter $E(z) = (N-1)Q(z)$ which is not particularly appropriate, since crosstalk is already absent and no specific control of the intersymbol interference is provided for. In Method 4, the form of the compensation filter should reduce to $E(z) = V(z^N)$ as no additional factor is necessary to cancel crosstalk. Then $V(z^N)$ will approximate the inverse of $Q(z)$. Note that for a general $Q(z)$ (not a function of z^N), using a compensation filter to suppress the mean-square intersymbol interference does not generally cancel the crosstalk.

In a subband system, the methods introduced in [10] to alleviate channel distortion are different from those proposed in this paper for a transmultiplexer. In fact, the presence of N channels in a subband system leads to a modification factor for the synthesis filters that is different for each synthesis filter [10]. This is in contrast to using one factor to modify each of the separation filters of a transmultiplexer. The modification approaches for both subband systems and transmultiplexers eliminate neither amplitude and phase distortion in the former nor intersymbol interference in the latter. Note that our methods 1, 2 and 3 are analogous to the approaches in [10]. In addition, we have proposed two additional procedures to control the intersymbol interference.

Table 1

Channel compensation filter and input-output transfer functions for the methods

Method	Compensation filter function	Input-output transfer function
1	$\prod_{i=1}^{N-1} Q(zW^{-i})$	$T(z^N)\prod_{i=0}^{N-1} Q(zW^{-i})$
2	$\dfrac{D(z)}{U_+(z)}\prod_{i=1}^{N-1} U_-(zW^{-i})$	$T(z^N)\prod_{i=0}^{N-1} U_-(zW^{-i})$
3	$\dfrac{D(z)}{U_+(z)}\dfrac{\prod_{i=1}^{N-1} U_-(zW^{-i})}{\prod_{i=0}^{N-1} U_-(z^{-1}W^{i})}$	$T(z^N)\prod_{i=0}^{N-1}\dfrac{U_-(zW^{-i})}{U_-(z^{-1}W^{i})}$
4	$V(z^N)\prod_{i=1}^{N-1} Q(zW^{-i})$	$T(z^N)V(z^N)\prod_{i=0}^{N-1} Q(zW^{-i})$
5	$W(z^N)\dfrac{D(z)}{U_+(z)}\prod_{i=1}^{N-1} U_-(zW^{-i})$	$T(z^N)W(z^N)\prod_{i=0}^{N-1} U_-(zW^{-i})$

3.2. Performance evaluation

We evaluate the performance of a transmultiplexer when the different channel compensation filters are used. Consider a two-band ($N=2$) transmultiplexer employing the Smith-Barnwell structure [9] such that $T(z^N)=N$ (an identity transmultiplexer). Now, the presence of a channel and a compensation filter introduces the extra term $S(z^N)$ in the input-output transfer function. By calculating $s(n)$ (the inverse z-transform of $S(z)$), we measure both the normalized peak distortion D_P and the normalized RMS distortion D_{RMS}. The normalized peak distortion is given by

$$D_P = \sum_{\substack{n \\ n\neq 0}} |s(n)|/|s(0)|. \tag{35}$$

The normalized RMS distortion is given by

$$D_{RMS} = \sqrt{\sum_{\substack{n \\ n\neq 0}} s^2(n) \Big/ s^2(0)}. \tag{36}$$

In many communications applications, the multiplexed output of the combining bank is converted to a lowpass analog signal (D/A conversion), modulated, sent across a channel and demodulated back to baseband. Then analog to digital (A/D) conversion takes place prior to the action of the separation bank. This process is equivalent to transmitting the lowpass analog signal over a lowpass equivalent channel and then performing A/D conversion as shown in Fig. 6. The D/A and A/D conversions are performed in phase and at the same sampling rate $f_s = 1/T_s$.

For our performance study, we need a discrete time equivalent $Q(z)$ that models the system of Fig. 6. The overall filtering effect of the D/A and A/D converters is assumed to be an ideal raised cosine filter with 50 percent roll-off. In the absence of a channel, the discrete time equivalent $Q(z)=1$. We consider a lowpass equivalent channel with a cubic phase characteristic (parabolic group delay) $\theta(\Omega)$ given by [7]

$$\theta(\Omega) = -\frac{\beta}{3\pi^2}(\Omega T_s)^3. \tag{37}$$

In effect, we are using a channel with a heavily distorted phase response that becomes more severe with increasing β. Such a phase nonlinearity exists over telephone channels and has been used to study the performance of multicarrier modems [4].

Specifically, we consider the discrete time equivalent response $q(n)$ for the case $\beta=5$. This is representative of the group delay distortion that is seen by a high speed modem over a telephone channel. The discrete time response $q(n)$ diminishes rapidly with $|n|$. An FIR $Q(z)$ with 43 coefficients spans the significant part of the response. The magnitude response of $Q(z)$ is flat up to the quarter sampling frequency and then decreases by 6 dB at the half sampling frequency due to aliasing effects. The delay is parabolic up to the quarter sampling frequency and then becomes more severe.

In calculating the normalized peak and RMS distortions for the first three methods, the reference coefficient that leads to the minimum distortion is aligned with the zeroth time index. This is equivalent to applying an additional time advance to the compensation filter. Although the impulse response is infinite for Method 3, lower bounds for the normalized peak and RMS distortions are computed by considering the first 60 samples. For Methods 4 and 5, the eigenvector corresponding to the minimum eigenvalue of the positive definite matrix is of dimension 61. Therefore, the component of the compensation filter involving the FIR least-squares approximation (denoted by $V(z^N)$ or $W(z^N)$) has 61 nonzero coefficients. In

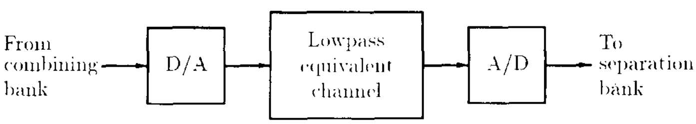


Fig. 6. Transmission over an analog channel.

addition, the coefficients of $V(z^N)$ or $W(z^N)$ are centered about the zeroth time index. This time index corresponds to the best reference coefficient of $s(n)$ without the least-squares filter.

The normalized peak and RMS distortions resulting from Methods 1, 2 and 3 are much larger than for Methods 4 and 5, primarily because there is no explicit suppression of the intersymbol interference. Specifically, Methods 1, 2 and 3 give peak distortions of 1.47, 0.46 and 1.11, respectively, and RMS distortions of 0.91, 0.37 and 0.58, respectively. Of the first three approaches, Method 2 achieves the lowest distortion and constrains the time span of the intersymbol interference. Method 3 is highly specific to preserving a stable IIR allpass transfer function. Even though the impulse response dies out with time, a large distortion results. Methods 4 and 5 are successful in that they result in very low peak and RMS distortions, all of which are below 10^{-4}.

4. Summary and conclusions

In this paper, new results for a multi-input, multi-output transmultiplexer are derived. Some are analogous to known results for a subband system. First, the issue of perfect reconstruction was explored. The multi-input, multi-output nature of a transmultiplexer generates a different set of input–output equations and hence a different condition for perfect reconstruction than for a subband system. The complementary nature of subband systems and transmultiplexers is interpreted through network duality and shown to be motivated by a sequence of block diagram manipulations that converts an identity subband system into an identity transmultiplexer. Since identity systems admit noncausal filters, they are converted into systems having an input–output transfer function equal to a pure delay by adding appropriate delay factors to the noncausal filter bank. In subband systems, we are free to choose any delay factor that results in a causal filter bank. However, in a transmultiplexer, the delay factor is constrained to be a multiple of the interpolation/decimation factor N.

Given that a transmultiplexer eliminates crosstalk, it is shown by both a direct approach and by network transposition that the set of combining and separation filters may be swapped, thereby preserving the crosstalk-free nature of the overall system. This is analogous to the swapping property in subband systems for the preservation of the alias-free nature.

In contrast to a subband system, the transmultiplexer has only one channel which is the medium of transmission for the composite signal formed from the various input data signals. In combatting channel effects, the principle is to use a single compensation filter that acts on the received composite signal prior to the action of the separation filter bank. This compensation filter is shown to have two components. One fixed component cancels crosstalk. The second component can be chosen to suppress intersymbol interference. Five choices for the second component are given. The first choice makes no attempt to control the intersymbol interference. Two other choices attempt to control either the time span of the intersymbol interference or the form of the input–output transfer function. The last two choices minimize the mean-square intersymbol interference. A performance evaluation involving a channel with a parabolic group delay shows that the last two choices achieve a low distortion.

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