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An improved and simplified design of cosine-modulated pseudo-QMF filterbanks

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Abstract

In this paper, a simple and efficient design of prototype filters for cosine-modulated filterbanks is proposed. Variable combinational window function with high side-lobe-fall-off-rate (SLFOR) has been used to design the prototype filter for cosine-modulated filterbanks. Cutoff frequency of the filter is optimized to minimize the reconstruction error, which is also selected as an objective function. Very small values of reconstruction and aliasing errors have been obtained with high SLFOR combinational window filters, resulting in near perfect reconstruction (NPR) filterbanks. However, higher filter order is required to get NPR solution.

Keywords: Filterbank; QMF; Cosine modulated

1. Introduction

Multirate filterbanks find applications in a wide variety of digital signal processing systems such as subband coding, transmultiplexing, image, video or audio compression, spectral estimation, and adaptive signal processing [1–4]. When the filterbanks are applied to these applications, it is desirable that the reconstructed output signal should be as close as possible to the input signal and the analysis and synthesis filters should have linear phase. Filterbanks with such properties have been studied in depth and various approaches have been successfully developed [2–12]. Johnston [5] introduced the first two-band, linear phase quadrature mirror filter (QMF) bank. Rothweiler [6] and Chu [7] independently developed M-band extensions to it. An M-band QMF (maximally decimated filterbank) bank consists of M parallel bandpass filters followed by M-fold down-sampler, while the corresponding synthesis bank has an M-fold up-sampler in each channel, followed by a bandpass filter and, finally, a summation of all the channels, as shown in Fig. 1. Among all M-band filterbanks, the cosine-modulated filterbank (CMFB) is one of the most frequently used filterbank because the design is simpler and more realizable than that of a general filterbank system [1,4,7–9]. All analysis and synthesis filters are cosine-modulated versions of a lowpass prototype filter. The design of whole filter-

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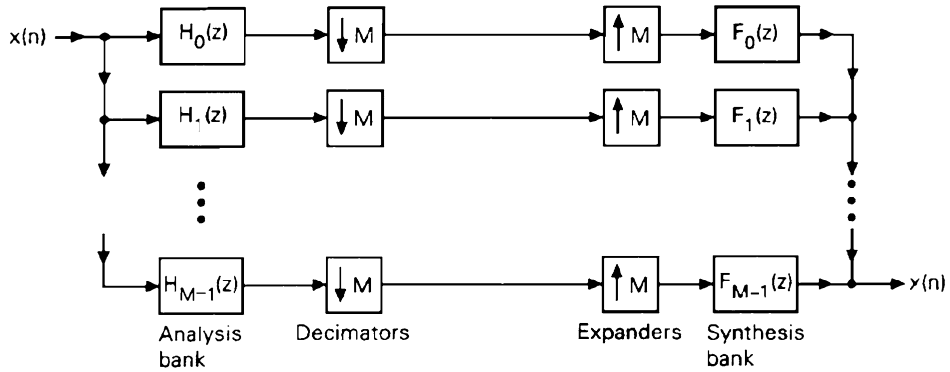


Fig. 1. M-band maximally decimated filterbank.

bank thus reduces to that of a single prototype filter. There are three sources of distortion in the reconstructed output of such filterbank—amplitude distortion, phase distortion, and aliasing [1]. All of the filterbanks rely on the flatness and linear phase of their prototype filters as well as on the orthogonality of their modulation functions to ensure good amplitude and phase reconstruction [2,3,12]. The filterbank has perfect reconstruction (PR), if the polyphase components of the prototype satisfy some pair-wise power complementary conditions [1,8]. For example, in [8–12], different methods for PR CMFBs are shown. However, this paper deals with a near perfect reconstruction (NPR) finite impulse response (FIR) cosine modulated filterbanks as they avoid the computation of large matrix sets, thereby reducing the computational burden during the implementation [13–16]. In these filterbanks, aliasing is canceled approximately and the distortion is only approximately a delay. Such approximate systems are called pseudo-QMF banks [1,16]. Let the prototype filter $P(e^{j\omega})$ be a linear phase. The conditions for approximate reconstruction can be stated in terms of $P(e^{j\omega})$ as follows [1,17]:

$$\left|P\left(e^{j\omega}\right)\right| \approx 0 \quad \text{for } |\omega| > \pi/M, \tag{1}$$

$$T\left(e^{j\omega}\right) \approx 1, \quad \text{where } T\left(e^{j\omega}\right) = \sum_{k=0}^{2M-1} \left|P\left(e^{j(\omega - k\pi/M)}\right)\right|^2. \tag{2}$$

The accuracy of the first approximation in turn gives a measure of the aliasing error and the accuracy of the second approximation gives a measure of the distortion error. Design approaches of the prototype filter involve nonlinear optimization as well as linear optimization [1,16,17]. Lin and Vaidyanathan [17] improvised the design method proposed in [16] by using Kaiser window. The optimization obtained in [17] has been achieved by using only a single parameter. Experimentally, they have shown that the objective function is a convex function of the parameter being optimized.

In this paper, a high SLFOR combinational window function is used in place of Kaiser window for designing prototype filter with reconstruction error selected as an objective function in place of the different objective function used by Lin and Vaidyanathan [17]. Cutoff frequency of the filter is optimized to achieve minimum value of reconstruction error. Reconstruction (E_{pp}) and aliasing errors (E_{a}) [1] are selected as the parameters to measure the performance of these filterbanks. A comparative performance analysis with the Kaiser window has also been done. Some of the results of this paper have been presented at recent conference [21].

2. Window approach

A filter $p(n)$ of length $(N+1)$ designed through window [18] is of the form:

$$p(n) = h_i(n)w(n), \quad \text{where } h_i(n) = \frac{\sin(\omega_{\mathrm{c}}(n - 0.5N))}{\pi(n - 0.5N)} \tag{3}$$

is the impulse response of the ideal filter with cutoff frequency ω_{c}, and $w(n)$ is the window function. Parzen-$\cos^6(n\pi/N)$ combinational window function (PC6) [19,20] given by (4) has been used to design the lowpass prototype filter for cosine-modulated filterbanks. The value of SLFOR for PC6 window function is -24 dB/octave, whereas this figure for Kaiser window is only −6 dB/octave [19]. The comparative study of the filters designed by using PC6 window has been made by Sharma et al. [20]. Combinational window functions are designed by combining a data window and a lag window in a linear manner. The expression for Parzen (lag window) and $\cos^6(n\pi/N)$ (data window) combinational window with γ as window shape parameter in discrete time domain is given as [19]

$$w_{\mathrm{PC6}}(n) = \begin{cases} \gamma[l(n)] + (1-\gamma)[d(n)], & |n| \leqslant N/2, \\ 0, & |n| > N/2, \end{cases} \quad 0 \leqslant \gamma \leqslant 3.7, \tag{4}$$

where

$$l(n) = \begin{cases} 1 - 24\left|\frac{n}{N}\right|^2\left(1 - 2\left|\frac{n}{N}\right|\right), & |n| < N/4, \\ 2\left(1 - 2\left|\frac{n}{N}\right|\right)^3, & N/4 \leqslant |n| \leqslant N/2, \end{cases}$$

and

$$d(n) = \cos^6\left(\frac{n\pi}{N}\right), \quad |n| \leqslant N/2.$$

The FIR filter design relationships for PC6 window are given by the following equations [20]:

(i) Relationship between window shape parameter (γ) and desired minimum stopband attenuation (Δ_s):

$$\gamma = a + (b\Delta_s) + \left(c\Delta_s^2\right), \tag{5}$$

where

$$\begin{aligned} &a = 8.15414, && b = -0.236709, && c = 0.00218617 && \text{for } 30.32 \leqslant \Delta_s \leqslant 51.25, \\ &a = 21.3669, && b = -0.605789, && c = 0.00434808 && \text{for } 51.25 < \Delta_s \leqslant 68.69. \end{aligned}$$

(ii) Relationship between normalized window width parameter (D) and (Δ_s):

$$D = a + (b\Delta_s) + \left(c\Delta_s^2\right), \tag{6}$$

where

$$\begin{aligned} &a = 1.82892, && b = -0.0275481, && c = 0.00157699 && \text{for } 30.32 \leqslant \Delta_s \leqslant 43.60, \\ &a = 1.67702, && b = 0.0450205, && c = 0.00000000 && \text{for } 43.60 < \Delta_s \leqslant 49.44, \\ &a = 85.4738, && b = -3.419690, && c = 0.03578400 && \text{for } 49.44 < \Delta_s \leqslant 57.48, \\ &a = -8.60006, && b = 0.4770040, && c = -0.00355655 && \text{for } 57.48 < \Delta_s \leqslant 68.69. \end{aligned}$$

A filter designed by the use of window is specified by three parameters—cutoff frequency (ω_c), filter order (N), and window shape parameter (γ). For desired stopband attenuation (Δ_s) and an appropriately chosen transition bandwidth ($\Delta\omega$), the order of the filter N can be estimated by

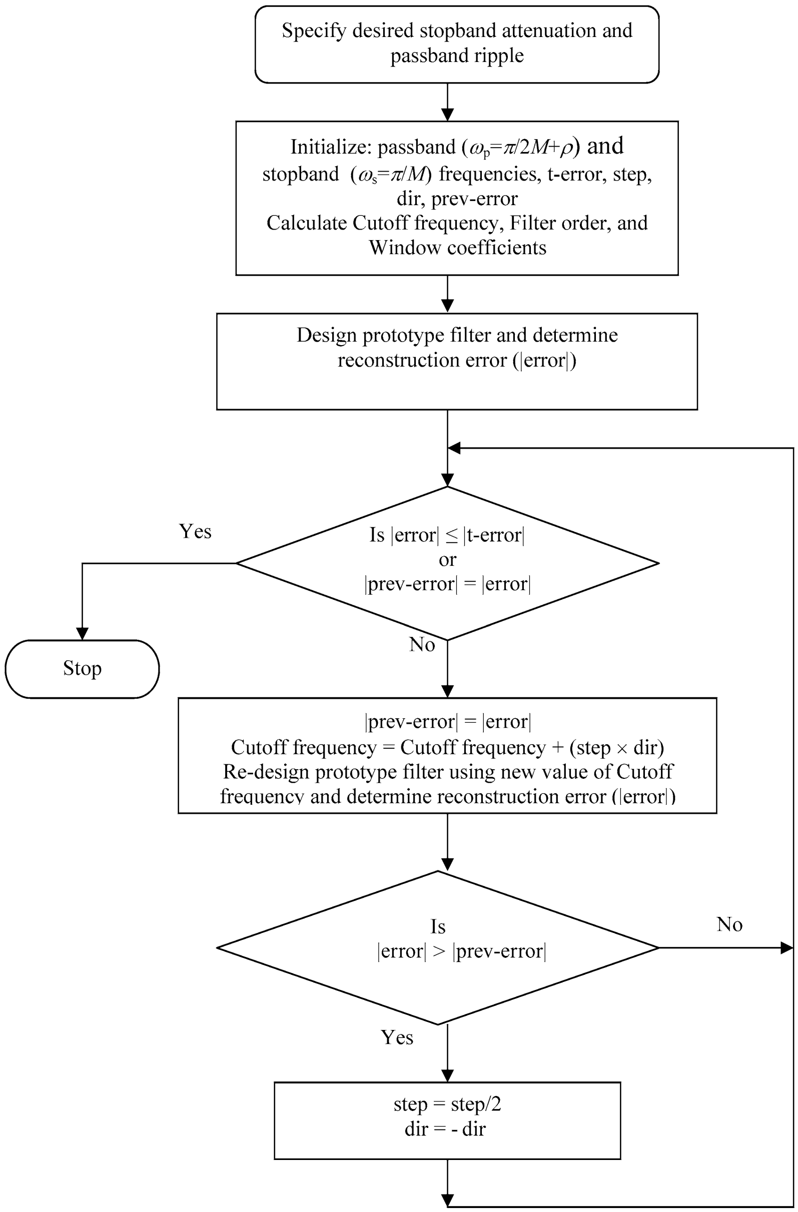
$$N \geqslant \frac{D}{\Delta\omega} + 1, \tag{7}$$

where $\Delta\omega$ is the normalized transition width $= (\omega_s - \omega_p)/2\pi$.

The parameter γ can be determined by the desired stopband attenuation using (5). After defining the window coefficients using (4), the cutoff frequency $\omega_c = 0.5(\omega_p + \omega_s)$ is the only parameter left in the filter design which can be optimized in such a way that the objective function given by following relationship is minimized.

$$\phi = \max_{\omega}\left|\left|P\left(e^{j\omega}\right)\right|^2 + \left|P\left(e^{j(\omega-\pi/M)}\right)\right|^2 - 1\right| \quad \text{for } 0 \leqslant \omega < \pi/M. \tag{8}$$

The algorithm developed in [16] and modified in [21] has been given in Scheme 1. Lin and Vaidyanathan used different objective function in [17] called ϕ_{new} and designed the prototype filters using Kaiser window. In this work, ϕ given by (8) has been used as an objective function in place of ϕ_{new} and improved designs have been obtained.



Scheme 1. Flowchart of the developed optimization algorithm.

3. Design example

For speech, 40 dB of attenuation in non-adjacent bands is sufficient to ensure an output signal of adequate quality [7,16]. Therefore, prototype filters with stopband attenuations in the range 40–60 dB in nonadjacent bands can provide an output signal of adequate quality.

In this example, 8-band cosine modulated filterbank has been designed using Parzen-$\cos^6(n\pi/N)$ and Kaiser window. The stopband attenuation of prototype filter is varied from 35 to 65 dB. The transition bandwidth is kept slightly smaller than $\pi/2M$ with stopband at π/M. The values of passband and stopband frequencies are fixed before calling the optimization algorithm. In this example, these values are 0.0545π and 0.1250π, respectively. The order of the prototype filter $P(e^{j\omega})$ is estimated by (7). At every iteration, the optimization algorithm adjusts the cutoff frequency ω_c, to minimize the objective function given by (8), without changing the values of other parameters. Filter order (N),

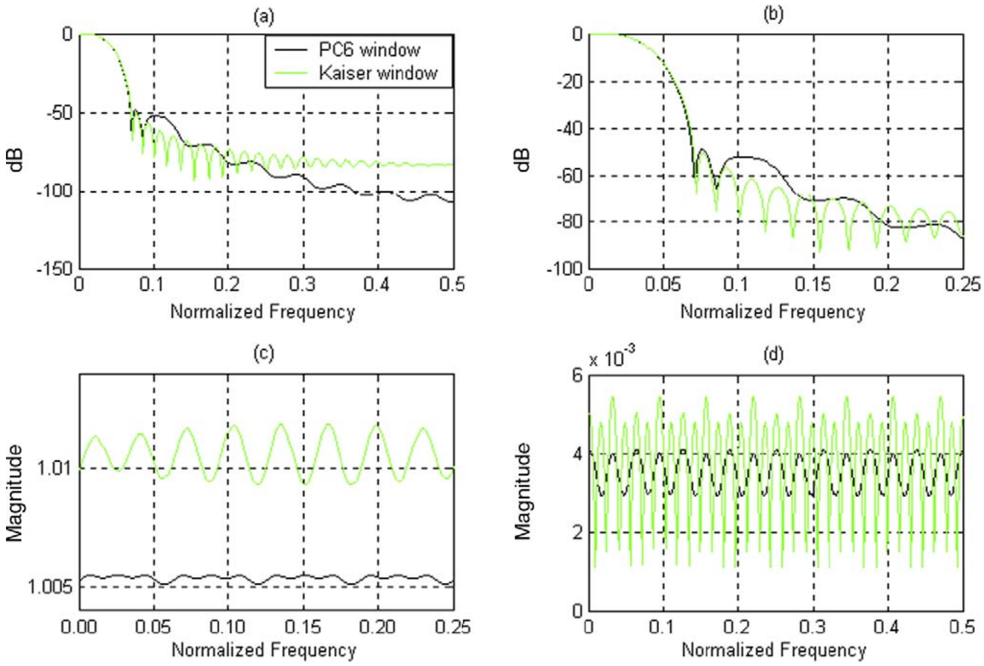


Fig. 2. (a) Magnitude of $P_0(e^{j\omega})$ (dB) for $\Delta_s = 50$ dB; (b) zoom plot of (a) for the interval $[0, 2\pi/M]$; (c) magnitude of $T(e^{j\omega})$ in the interval $[0, 2\pi/M]$, as $T(e^{j\omega})$ is periodic; (d) aliasing error.

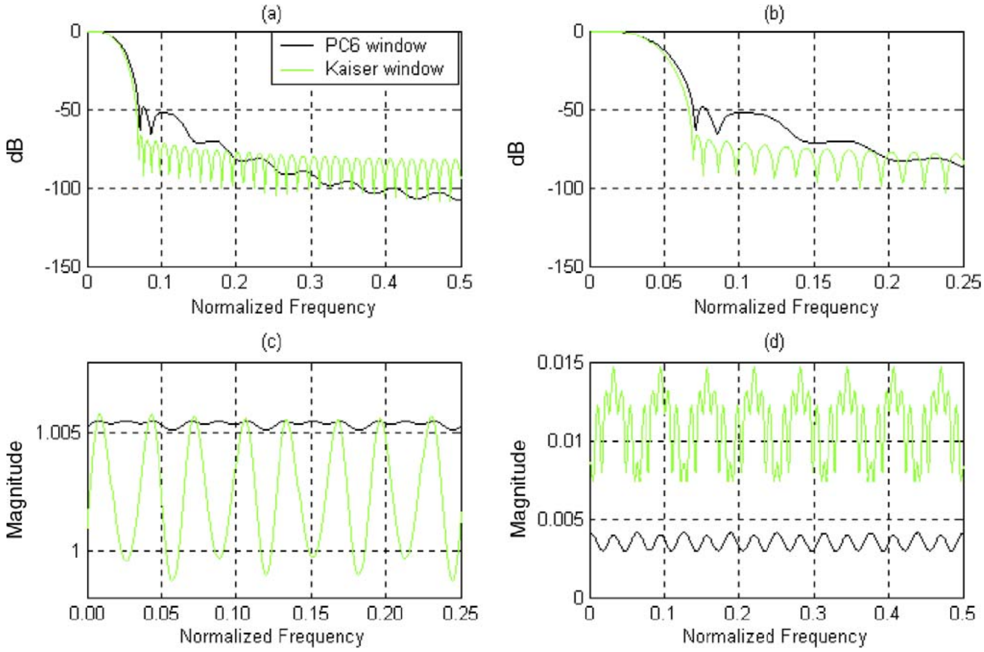


Fig. 3. (a) Magnitude response of $P_0(e^{j\omega})$ (dB) for $N = 68$; (b) zoom plot of (a); (c) magnitude of $T(e^{j\omega})$ in the interval $[0, 2\pi/M]$; (d) aliasing error.

peak-to-peak reconstruction error (E_{pp}) and aliasing error (E_a) have been selected as performance measuring parameters [1]. Values of these parameters for Kaiser window and PC6 window prototype filters are calculated. Figures 2 and 3 show the comparative performance of the Kaiser window and PC6 window based filterbanks. The response of the prototype filters shown in Figs. 2a and 3a are optimized using the developed algorithm. In Fig. 2, stopband attenu-

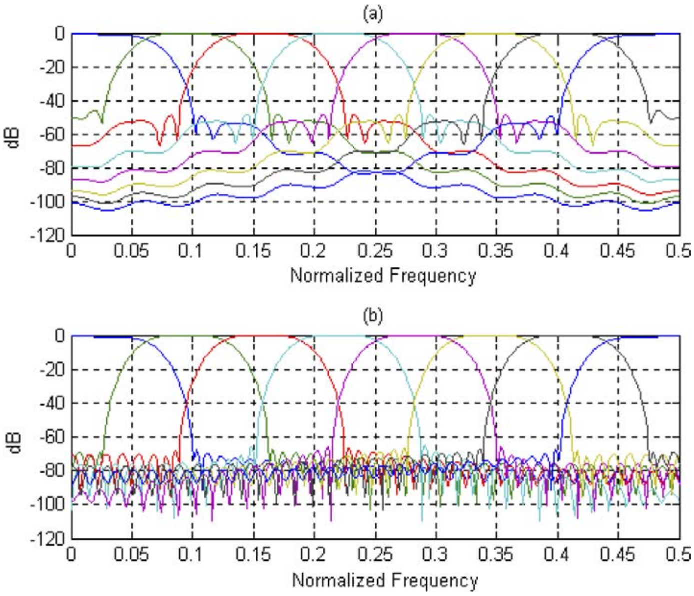


Fig. 4. Magnitude plot of 8-band cosine-modulated filterbank using: (a) PC6 window; (b) Kaiser window.

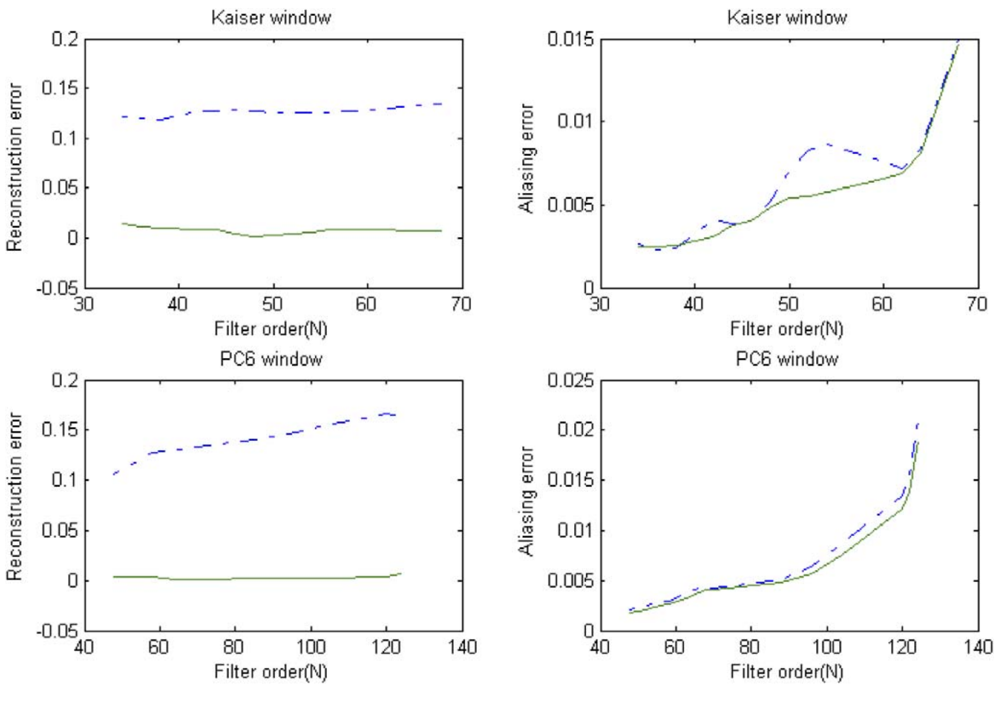


Fig. 5. Plots of filter-order vs reconstruction error and filter-order vs aliasing error. (—) Proposed method. (- - -) As per Lin and Vaidyanathan [17].

ation Δ_s is fixed at 50 dB, resulting in values of N equal to 50 and 68 for Kaiser and PC6 window filters, respectively. A zoom plot of these prototypes is shown in Fig. 2b. Figures 2c and 2d illustrate the reconstruction and aliasing errors, respectively. Figure 3 shows a set of similar comparative plots. Here filter order (N) is fixed at 68, resulting in values of stopband attenuation Δ_s equal to 65 and 50 dB for the Kaiser and PC6 prototype filters, respectively. These figures clearly show the superiority of PC6 window over Kaiser window, because smaller values of reconstruction and aliasing errors are obtained with PC6 window. Although lower stopband attenuation is obtained in case of PC6 window for the same filter order. Figures 4a and 4b show the magnitude response of the 8 band PC6 window and Kaiser window filterbank, respectively for $N = 68$. Also, as is evident from Fig. 5, the proposed algorithm outclasses the performance of the algorithm used by Lin and Vaidyanathan [17] in terms of obtained values of reconstruction errors.

4. Conclusions

A simple algorithm has been proposed to design window based prototype filters for cosine-modulated filterbanks. Simulation studies shows that the developed algorithm provides filterbanks with very small values of reconstruction and aliasing errors. An improvisation in the performance of filterbank is obtained by using high SLFOR window over Kaiser window for designing prototype filters. The PC6 window offers the advantage of a smaller reconstruction error and a smaller aliasing error at the expense of reduced stopband attenuation. The algorithm is very simple and takes equally better execution time. These pseudo-QMF filterbanks can be used in real time applications such as echo cancellation and cross-talk suppression with minimum aliasing and reconstruction errors at no additional cost.

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References

[1] P.P. Vaidyanathan, Multirate Systems and Filter Banks, Prentice Hall, Englewood Cliffs, NJ, 1993.
[2] T. Nagai, T. Fuchie, M. Ikehara, Design of linear phase M-channel perfect reconstruction FIR filter banks, IEEE Trans. Signal Process. 45 (9) (1997) 2380–2387.
[3] P. Saghizadeh, A.N. Willson, A new approach to the design of critically sampled M-channel uniform-band perfect-reconstruction linear-phase FIR filter banks, IEEE Trans. Signal Process. 46 (6) (1998) 1544–1557.
[4] F. Cruz-Roldan, M. Monteagudo-Prim, Efficient implementation of nearly perfect reconstruction FIR cosine-modulated filterbanks, IEEE Trans. Signal Process. 52 (9) (2004) 2661–2664.
[5] J.D. Johnston, A filter family designed for use in quadrature mirror filter banks, in: Proc. IEEE Int. Conf. Acoustics, Speech, Signal Process., 1980, pp. 291–294.
[6] J.H. Rothweiler, Polyphase quadrature filters: A new subband coding technique, in: Proc. IEEE Int. Conf. Acoustics, Speech, Signal Process., Boston, MA, 1983.
[7] P.L. Chu, Quadrature mirror filter design for an arbitrary number of equal bandwidth channels, IEEE Trans. Acoust. Speech Signal Process. ASSP-33 (1) (1985) 203–218.
[8] R.D. Koilpillai, P.P. Vaidyanathan, Cosine-modulated FIR filterbanks satisfying perfect reconstruction, IEEE Trans. Signal Process. 40 (4) (1992) 770–783.
[9] P.N. Heller, T. Karp, T.Q. Nguyen, A general formulation of modulated filter banks, IEEE Trans. Signal Process. 47 (4) (1999) 986–1002.
[10] X. Gao, Z. He, X.-G. Xia, Efficient implementation of arbitrary-length cosine-modulated filter banks, IEEE Trans. Signal Process. 47 (4) (1999) 1188–1192.
[11] X. Gao, Z. He, X.-G. Xia, The theory and implementation of arbitrary-length linear-phase cosine-modulated filter bank, Signal Process. 80 (5) (2000) 889–896.
[12] T. Karp, A. Mertins, G. Schuller, Efficient biorthogonal cosine-modulated filter banks, Signal Process. 81 (5) (2001) 997–1016.
[13] T.Q. Nguyen, Near-perfect-reconstruction pseudo-QMF banks, IEEE Trans. Signal Process. 42 (1) (1994) 65–76.
[14] R.D. Koilpillai, P.P. Vaidyanathan, A spectral factorization approach to pseudo-QMF design, IEEE Trans. Signal Process. 41 (1) (1993) 82–92.
[15] F. Cruz-Roldan, P. Amo-Lopez, P. Martin-Martin, Lopez-Ferreras, Alternating analysis and synthesis filters: A new pseudo-QMF bank, Digital Signal Process. 11 (4) (2001) 329–345.
[16] C.D. Creusere, S.K. Mitra, A simple method for designing high-quality prototype filters for M-band pseudo-QMF banks, IEEE Trans. Signal Process. 43 (4) (1995) 1005–1007.
[17] Y. Lin, P.P. Vaidyanathan, A Kaiser window approach for the design of prototype filters of cosine-modulated filterbanks, IEEE Signal Process. Lett. 5 (6) (1998) 132–134.
[18] A.V. Oppenheim, R.W. Schafer, Discrete-Time Signal Processing, Prentice Hall, Englewood Cliffs, NJ, 1989.
[19] R. Saxena, Synthesis and characterization of new window families with their applications, Ph.D. thesis, Electronics and Computer Engineering Department, University of Roorkee, Roorkee, India, 1997.
[20] S.N. Sharma, R. Saxena, S.C. Saxena, Design of FIR filters using variable window families: A comparative study, J. Ind. Inst. Sci. 85 (5) (2004) 155–161.
[21] A. Jain, R. Saxena, S.C. Saxena, An improved and simplified design of prototype filters for cosine-modulated filterbanks using Kaiser window, in: Proc. 8th World Multiconference on Systemics, Cybernetics and Informatics (SCI 2004), vol. VI, Orlando, USA, 2004, pp. 145–147.

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